[Year]

DIFFRACTION-Study Material

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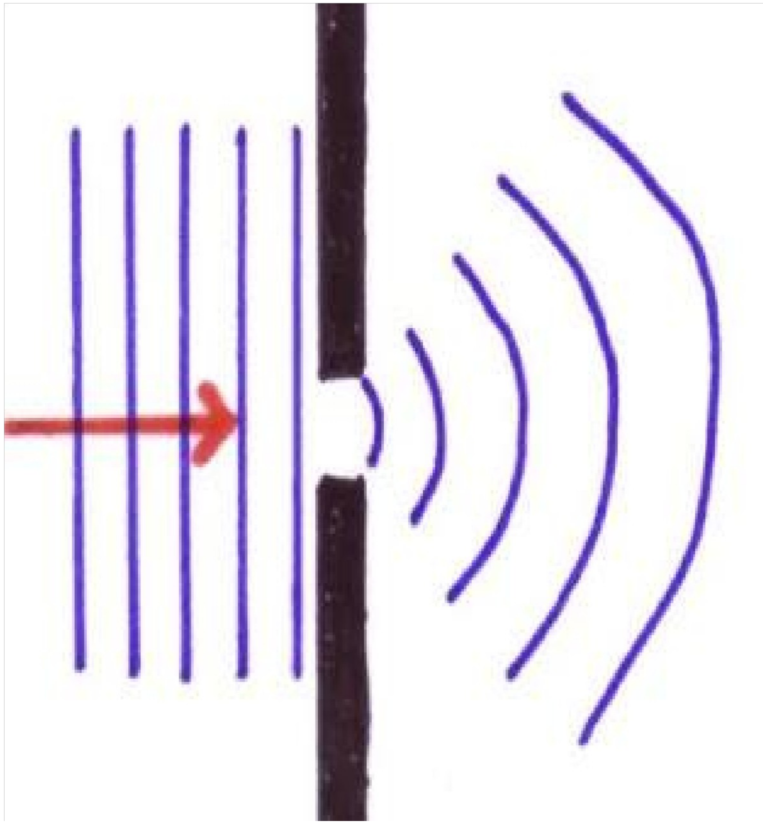
Diffraction

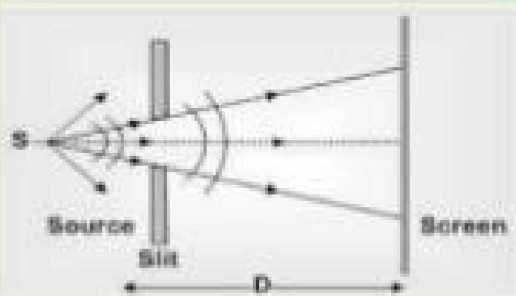
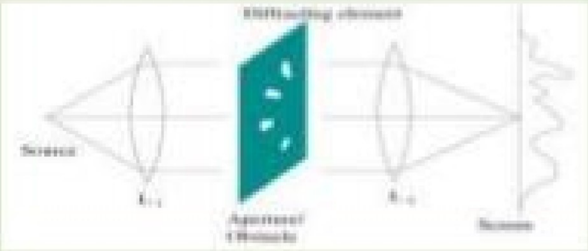
lights behave like waves. If light waves encounter obstacles or a slit with a small gap, waves start propagating from that gap. That gap or diffracting aperture becomes the secondary source of wave propagating wave.



Gap in the obstacles behaves like a secondary source of the wave because light bends around a corner or obstacle and this phenomenon of a wave is called Diffraction.

Classification of Diffraction

Diffraction phenomena of light can be divided into two different classes

Fresnel's Diffraction	Fraunhofer diffraction
	
Cylindrical wave fronts	Planar wave fronts
Source of screen at finite distance from the obstacle	Observation distance is infinite. In practice, often at focal point of lens.
Move in a way that directly corresponds with any shift in the object.	Fixed in position
Fresnel diffraction patterns on flat surfaces	Fraunhofer diffraction patterns on spherical surfaces.
Change as we propagate them further ‘downstream’ of the source of scattering	Shape and intensity of a Fraunhofer diffraction pattern stay constant.

Fraunhofer diffraction	Fresnel diffraction
Source and the screen are far away from each other.	Source and screen are not far away from each other.
Incident wave fronts on the diffracting obstacle are plane.	Incident wave fronts are spherical.
Diffracting obstacle give rise to wave fronts which are also plane.	Wave fronts leaving the obstacles are also spherical.
Plane diffracting wave fronts are converged by means of a convex lens to produce diffraction pattern.	No Convex lens is needed to converge the spherical wave fronts.

FRESNEL'S ASSUMPTIONS:

i) Each element of a wave-front sends secondary waves continuously.

ii) A wave front can be divided into a large number of strips or zones called the Fresnel's zones

ii) The resultant effect at any external point P is determined by combining the effects of all the secondary waves reaching them from various zones.

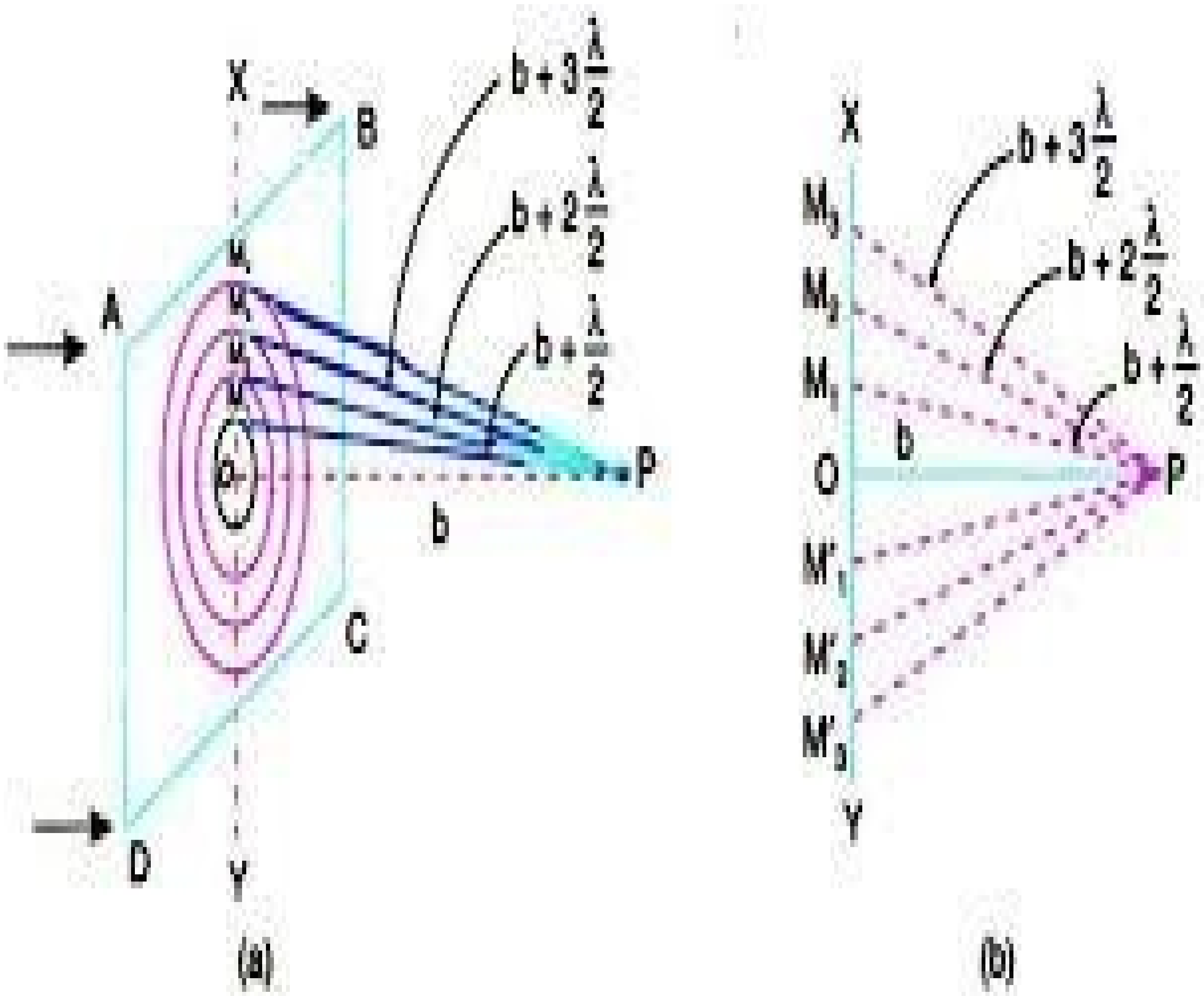
iv) The effect at a point due to any particular zone depends on the distance of the point from the zone.

v) The effect at any point P depends on the inclination of the point with reference to zone under consideration. The intensity is maximum along a direction normal to the zone and decreases as the angle of inclination Increases.

FRESNEL'S HALF PERIOD FOR A PLANE WAVE :

- Let ABCD be a section of a plane wave-front of monochromatic light of wave length , traveling from left to right (fig). Let P be an external point at which the effect of the entire wave-front is desired.

Let OP is perpendicular drawn from P to the wave-front and is equal to b. To find the resultant intensity at P due to the wave-front, by Fresnel's method, the wave-front is divided In to a number of concentric half period zones called "Fresnel's zone." and then the effect of all the zones at point p is found. This can be done as follows: Considering P as centre and radii equal to b+λ/2, b+2$\lambda/2$, b+3$\lambda/2$,... etc. draw a series of spheres on the wave-front thus cutting the wave-front into annular strips or zones. The sections of these spheres by the plane wave-front are concentric circles having common centre O and radii $0M^1$, $0M^2$, $0M^3$...$0M^{n-1}$, $0M^n$ ......etc.



X
B
A
C
D
Y
P
b
b + 3λ/2
b + 2λ/2
b + λ/2
(a)
X
M3
M2
M1
O
M'1
M'2
M'3
Y
P
b
b + 3λ/2
b + 2λ/2
b + λ/2
(b)

The secondary wavelets from any two consecutive zones reach P with a path difference $\lambda/2$ or time difference half period. That is why the zones are called half period zones. THE area of the innermost (first) circle is called first half period zone; similarly the annular area between the first circle and the second circle is called second half period zone and so on. Thus the annular area between $(n-1)^{th}$ and nth circle is called nth half period zone.

The point O is called the pole of the wave-front with respect to point P. A Fresnel half period zone with respect to an external point P is a thin annular zone or a thin strip of the primary wave-front surrounding the point O such that the distances of its outer and inner edges from O differs by $\lambda/2$.

RADII OF HALF PERIOD ZONE:

The radius of first half period zone,

$$OM_1 = \sqrt{(M_1P)^2 - (OP)^2}$$

$$OM_1 = \sqrt{\left(b + \frac{\lambda}{2}\right)^2 - b^2}$$

$$OM_1 = \sqrt{b^2 + \frac{\lambda^2}{4} + b\lambda - b^2}$$

$$OM_1 = \sqrt{b\lambda}\ , \text{As } b >> \lambda$$

The radius of second half period zone,

$$OM_2 = \sqrt{(M_2P)^2 - (OP)^2}$$

$$OM_2 = \sqrt{\left(b + \frac{2\lambda}{2}\right)^2 - b^2}$$

$$OM_2 = \sqrt{b^2 + \frac{4\lambda^2}{4} + 2b\lambda - b^2}$$

$$OM_2 = \sqrt{2b\lambda}$$

Similarly the radius of the nth half period zone is,

$$OM_n = \sqrt{(M_nP)^2 - (OP)^2}$$

$$OM_n = \sqrt{\left(b + \frac{n\lambda}{2}\right)^2 - b^2}$$

$$OM_n = \sqrt{b^2 + \frac{(n\lambda)^2}{4} + nb\lambda - b^2}$$

$$OM_n = \sqrt{nb\lambda}$$

Thus we see that the radii of half period zones are proportional to the square root of the natural numbers.

Area of a half period zone:

The area of n zone

$$= \pi[(OM_n)^2 - (OM_{n-1})^2]$$
$$= \pi[(PM_n)^2 - (PO)^2] - \pi[(PM_{n-1})^2 - (PO)^2]$$

$$= \pi\left[\left(b + \frac{n\lambda}{2}\right)^2 - (b)^2\right] - \pi\left[\left(b + \frac{(n-1)\lambda}{2}\right)^2 - (b)^2\right]$$
$$= \pi\left[b^2 + \frac{[n\lambda]^2}{4} + bn\lambda - b^2\right] - \pi\left[b^2 + \frac{[(n-1)\lambda]^2}{4} + b[n-1]\lambda - b^2\right]$$

$$= \pi\left[\frac{[n\lambda]^2}{4} + bn\lambda\right] - \pi\left[\frac{[(n-1)\lambda]^2}{4} + b[n-1]\lambda\right]$$
$$= \pi\left[n\lambda + \frac{\lambda^2}{4}(2n-1)\right]$$
$$= \pi b\lambda$$

- As area of nth zone is independent of n, thus the area of each half period zone is approximately the same and is equal More accurately the area of the zone increases slightly with n.

THE DISTANCE OF THE POINT FROM THE HALF PERIOD ZONE:

The average distance of n^{th} half period zone from point P,

$$= \frac{\left[b + \frac{n\lambda}{2}\right] + +b + (n-1)\frac{\lambda}{2}}{2} = b + (2n-1)\frac{\lambda}{4}$$

THE AMPLITUDE OF THE DISTURBANCE AT P DUE TO AN INDIVIDUAL ZONE :

The amplitude of the disturbance due to a given zone is

(i) Directly proportional to the area of the zone because number of point sources, from each of which a secondary wavelet starts, in a zone are proportional to the area,

(ii) Inversely proportional to the distance of the point P from the given zone.

(iii) Directly proportional to the obliquity factor $(1+\cos\theta)$ whose θ is the angle between the normal to the zone and the line joining the zone to point P.

Thus amplitude of the disturbance at P due to n^{th} zone is,

$$R_n \alpha \frac{\pi\left[b\lambda + \frac{\lambda^2(2n-1)}{4}\right]}{b + \frac{(2n-1)\lambda}{4}}(1 - cos\theta_n)$$

$$R_n \alpha\ \pi\lambda(1 - cos\theta_n)$$

As n increases, θ_n increases and cos θ_n decreases. Thus the amplitude of the disturbance at P due to a given zone decreases as the order of the zone increases. This means that the amplitude of the disturbance due to first half period zone is maximum and it decreases regularly as we pass from the inner zone to the next outer.

THE RESULTANT AMPLITUDE DUE TO THE WHOLE WAVE-FRONT:

Let $R_1, R_2, R_3, \ldots\ldots R_n$ be the amplitudes of the disturbances at P, due to the first second, third...., nth half period zones respectively. The magnitudes of R_1, R_2 etc are of continuously in decreasing order. As the path difference between the wave reaching P from any two consecutive half-period zones is $\lambda/2$, the waves from two consecutive zones reach P in the opposite phase.

Therefore if amplitude due to first zone is positive, that due to second zone is negative, that due to third zone is positive and so on, i e $R_1, R_3,$ …..R_{n-1} etc are positive and R_2, R_4, …..R_n etc, are negative Hence the resultant amplitude at P due to the entire wave front is

$$R = R_1 - R_2 + R_3, \dots (-1)^{n-1} R_n$$

As the magnitudes of successive terms R_1, R_2, R_3, …..R_n decrease gradually, R_2slightly **less** than R_1 and greater than R_3 so that we may write,

$$R_2 = \frac{R_1 + R_3}{2}$$

$$R_3 = \frac{R_2 + R_5}{2}$$

And so on.

Now equation may be written in the form

$$R = \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \left(\frac{R_3}{2} - R + \frac{R_5}{2}\right) + \dots\dots\dots \frac{R_n}{2}$$

if n is odd.

And

$$R = \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \left(\frac{R_3}{2} - R_3 + \frac{R_5}{2}\right) + \dots\dots\dots \frac{R_{n-1}}{2} - R_n$$

if n is even.

In above relations the quantities in the bracket is very nearly equal to zero hence we can write

$$R = \frac{R_1}{2} + \frac{R_n}{2} \quad if\ n\ is\ odd$$

$$R = \frac{R_1}{2} + \frac{R_n}{2} \quad if\ n\ is\ even$$

But usually n is large, hence we may write R_{n-1} (appro).then

$$\frac{R_{n-1}}{2} - R_n = -\frac{R_n}{2}$$

So that above two equations may be represented by a single equation as

$$R = \frac{R_1}{2} \pm \frac{R_n}{2}$$

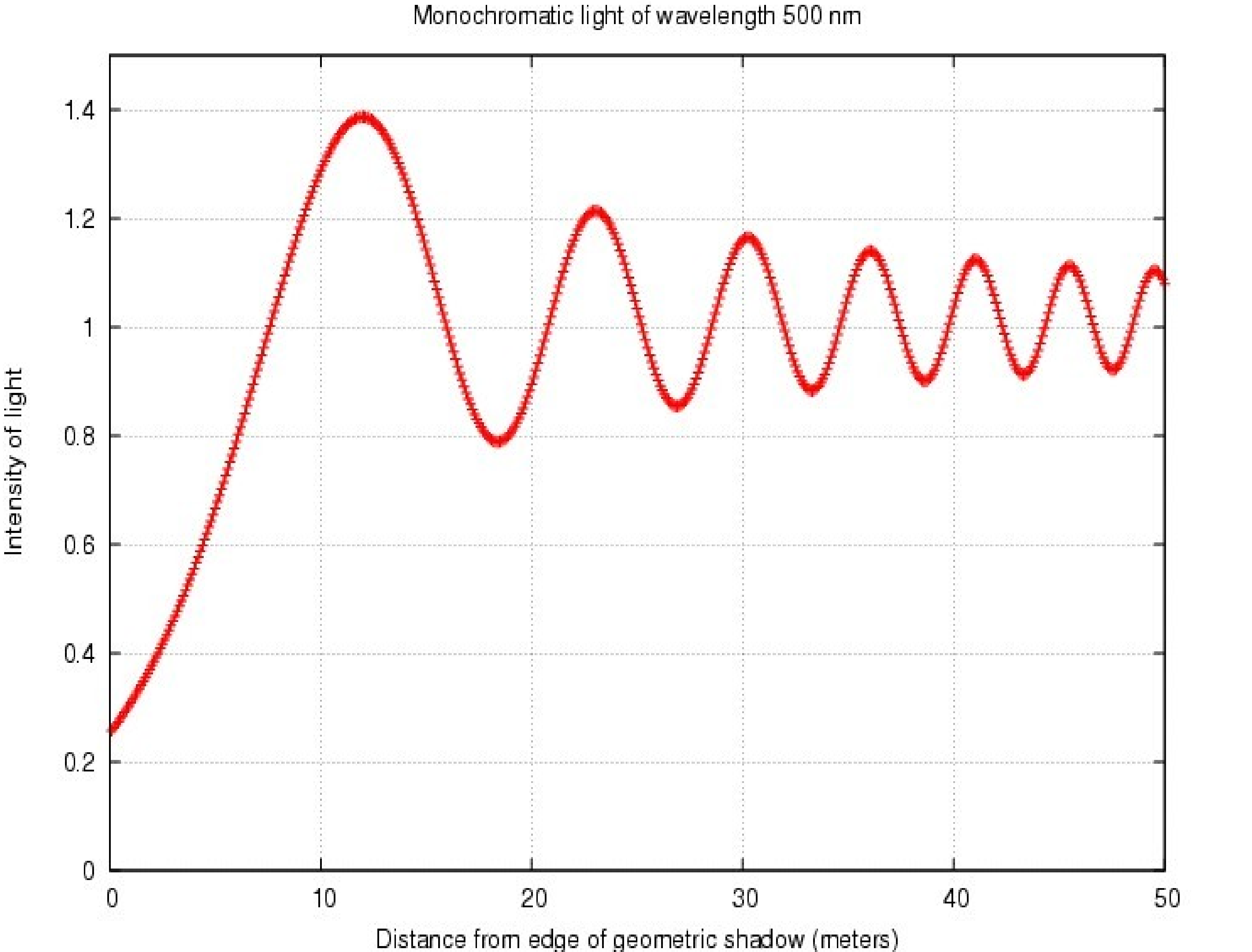
The plus and minus sign being taken accordingly as n is odd or even. For large wave front, n is very large, hence Rn vanishes; so that we can have

$$R = \frac{R_1}{2}$$

Thus the amplitude due to a large wave front at a point in front of it is half that due to the first half period zone acting alone. The intensity at any point is proportional to the square of the amplitude, therefore the resultant Intensity at P

$$I\alpha \frac{R_n}{4}$$

This means the intensity at appoint is one forth of intensity due to the first half period zone alone.



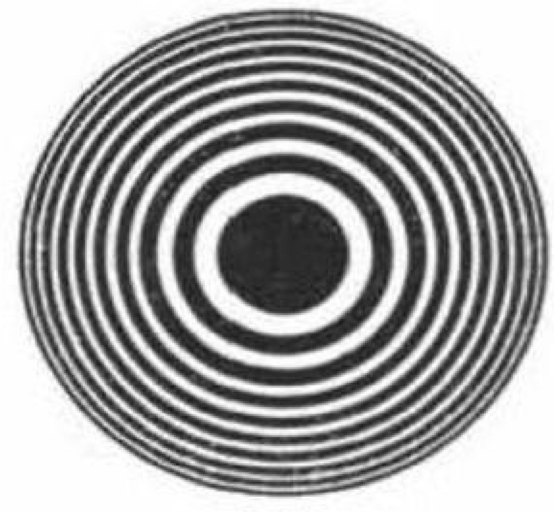
Monochromatic light of wavelength 500 nm
Intensity of light
Distance from edge of geometric shadow (meters)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
10
20
30
40
50

Zone Plate

This is a special screen designed to block off the light from every other half-period zone. The result is to remove either all positive terms in eq. (1) or all negative terms. In either case the amplitude at P will be increased to many times in value in the above cases.

(a)

(Positive Zone plate)



(b)

(Negative Zone plate)

Principle The resultant intensity at a point is

$$A = A_1 - A_2 + A_3 - A_4 + \ldots\ldots + (-1)^{m-1} A_m \ldots.(1)$$

If n very large then $A = \frac{A_1}{2}$

Intensity at P is $I = \frac{A_1^2}{4}$

If the even numbered zones are blocked

$$A = A_1 + A_3 + A_5 \ldots\ldots\ldots.(2)$$

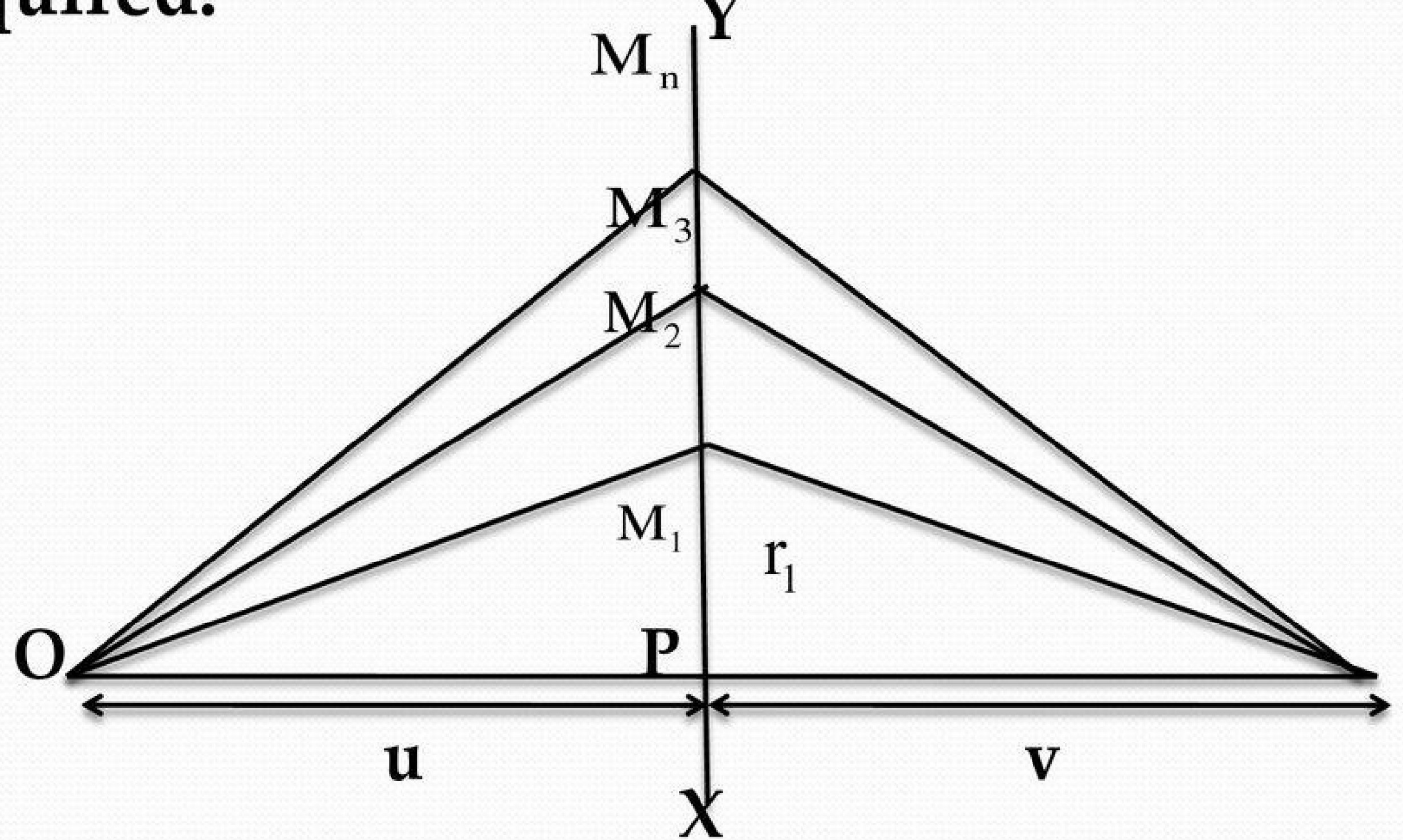
If the odd numbered zones are blocked

$$A = -(A_2 + A_4 + A_5 ......)....(3)$$

The magnitude of amplitude A in both the cases (2) and (3) will almost be the same and the net illumination at the point P (intensity proportional to A^2) will be very very large.

Construction **Draw circles on a white paper sheet, with radii proportional to the square root of natural numbers. The odd or even numbered zones are covered with ink. Take photograph of this pattern on a thin glass plate on a reduced scale. This form ZONE PLATE**

Theory of zone plate let XY be the section of zone plate perpendicular to the plane of paper. Let O be the luminous object emitting spherical waves of wavelength λ whose effect at point I is required.



Let u be the distance of the object O from the zone plate and v is the distance of the screen from the zone plate. $PM_1.PM_2,\ldots$ ($r_1, r_2, r_3 \ldots r_n$) are the radii of first , second …etc half period zones. Position of the screen is such that from one zone to the next, there is increase on path difference of λ/2.

$$OP + PI = u + v$$

$$OM_1 + M_1 I = u + v + \frac{\lambda}{2}$$

$$OM_2 + M_2 I = u + v + \frac{2\lambda}{2}$$

And so on

$$OM_n + M_n I = u + v + \frac{n\lambda}{2} \ldots(4)$$

Similarly points exists on the section XY of ZP below the pole P at exactly the same distances.

$$OM_n = \sqrt{OP^2 + PM_n^2}$$

$$= (u^2 + r_n^2)^{\frac{1}{2}} = u\left(1 + \frac{r_n^2}{u^2}\right)^{\frac{1}{2}}$$

$$OM_n = u\left(1 + \frac{1}{2}.\frac{r_n^2}{u^2}\right) \Rightarrow OM_n = u + \frac{r_n^2}{2u}$$

Similarly $\quad M_nI = v + \dfrac{r_n^2}{2v}$

Substituting these in (4)

$$u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + \frac{n\lambda}{2}$$

$$\frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{n\lambda}{2}$$

$$\pi(r_n^2 - r_{n-1}^2) = \frac{\pi\lambda uv}{u+v}[n-n+1] = \frac{\pi\lambda uv}{u+v} = \pi\lambda f$$

this is independent of n. hence for a given object O and I, the area of all the zones remain the same. Area diminishes as u and v decreases i.e. plate is approached by the object or image.

Working of the zone plate as a lens.

$$A = A_1 - A_2 + A_3 - A_4 + \dots\dots\dots(1)$$

$$A = \frac{A_1}{2}$$

$$A = A_1 + A_3 + A_5 ....(2)$$

$$\frac{1}{u} + \frac{1}{v} = \frac{n\lambda}{r_n^2} ....(5)$$

This result is similar to the lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} ....(6)$$

From (5) and (6)

$$\frac{1}{f} = \frac{n\lambda}{r_n^2}$$

$$f = f_n = \frac{r_n^2}{\lambda n} ....(7)$$

This eqn. shows that a zone plate has a number of foci which depend on the number of zones as well as wavelength of light used.

Multiple foci of a zone plate the focal length of a zone plate is

$$f = f_n = \frac{r_n^2}{\lambda n} \text{........}(7)$$

$$n = 1,2,3,4\text{........}$$

Thus it behaves like a convex lens, but it has a multiple foci between the zone and point I. this is because the **number of half period zones contained in an area depend upon the position of the screen.**

For n = 1

$$f = f_1 = \frac{r_1^2}{n\lambda} \text{........}(8)$$

Say screen at point I obtained using this formula is called the **first order focal point (or as primary or principle focal length). Area of each zone is $\pi\lambda f$.**

Resultant amplitude is given by

$$A = A_1 + A_3 + A_5 ....(2)$$

and the intensity is most intense.

Now for n = 2

let the screen is at I_2 such that the distance PI_2= f/2 area of each zone is =$\pi\lambda$f/2. Now each zone for I will now contain 2 zones for I_2. Hence for I_2, 1st and 2nd zones will be exposed and 3rd and 4th zones are blackened.

$$A = (A_1 - A_2) + (A_5 - A_6) + (A_9 - A_{10}) + ...$$

Since $A_1 \approx A_2$ **and** $A_5 \approx A_6$ **so on therefore**

$$A = 0$$

Now for n = 3

let the screen is at I_3 such that the distance $PI_3 = f/3$ area of each zone is $= \pi\lambda f/3$. therefore, focal length is

$$f_3 = \frac{r_1^2}{3n\lambda}$$

Now each zone for I will now contain 3 zones for I_3. Hence for I_3, 1st , 2nd and 3rd zones will be exposed and 4th, 5th and 6th zones are blackened.

$$A = (A_1 - A_2 + A_3) + (A_7 - A_8 + A_9) + \ldots\ldots$$

$$A = \left(\frac{A_1}{2} + \underbrace{\frac{A_1}{2} - A_2 + \frac{A_3}{2}}_{0} + \frac{A_3}{2}\right) + \left(\frac{A_7}{2} + \underbrace{\frac{A_7}{2} - A_8 + \frac{A_9}{2}}_{0} + \frac{A_9}{2}\right) + \ldots\ldots$$

$$A = \frac{A_1}{2} + \frac{A_3}{2} + \frac{A_7}{2} + \frac{A_9}{2} + \ldots\ldots$$

Which shows that amplitude at I_3 is greater than $\frac{A_1}{2}$

i.e. light is focused at I_3 although its intensity is less than that at I. hence this point is also called as focal length. As $f_3 = f/3$, it is rightly called **third order focal length**. Similarly it can be shown that

$$f_5 = \frac{r_1^2}{5n\lambda}, f_7 = \frac{r_1^2}{7n\lambda}, f_9 = \frac{r_1^2}{9n\lambda}$$

Also, the light focuses though intensity or brightness goes on decreases. They are all known **as focal lengths**. Hence, unlike lens, zone plate has **multiple foci.**

Focusing action of zone plate

When a zone plate is placed in front of source S, on the other side of the axis of the plate, we observe a series of points with their intensities in increasing order as the distance of points from plate increases. Thus the zone plate acts as a conversing lens. However, it is different from a lens, because it is associated with a series of foci and focal lengths.

dissimilarities

a) **The zone plate works by diffraction and the lens works by refraction**

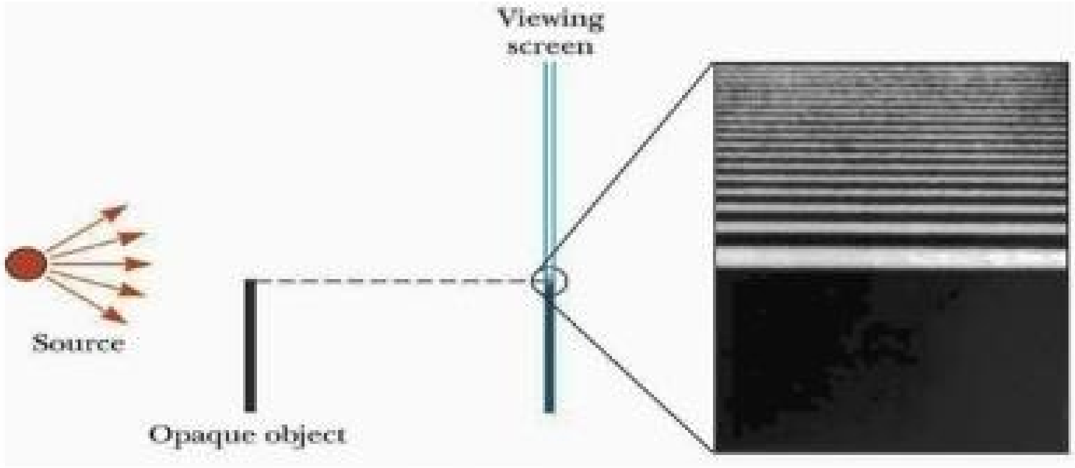
b) **The image produced by a lens is very intense where as the image produced by the zone plate not intense.**

c) **The zone plate has an array of focal lengths (N numbers) where as a convex lens has a single focal point.**

d) **The focal length of a convex lens directly proportional to the wavelength where as the focal length of the zone plate inversely proportional to wavelength.**

e) There is no time delay when light is passing through one point to another in the convex lens but there is a time delay as light passes from one period to other in zone plate.

Diffraction Pattern, Object Edge



- This shows a ***diffraction pattern*** associated with light from a single source passing by the ***edge of an opaque object***
- The diffraction pattern ***is vertical*** with the central maximum at the bottom

Positions of Max and Min Intensity

Let $SO = a$; $OP = b$; $PQ = x$

P.D bet the Vibration from O' and O

$$\delta = OQ - O'Q$$

$$OQ = \sqrt{b^2 + x^2} = b\left(1 + \frac{x^2}{b^2}\right)^{1/2}$$

$$= b + \frac{x^2}{2b} \quad \because x \ll b$$

$|||^{ly}$

$$O'Q = SQ - SO'$$

$$SQ = \sqrt{(a+b)^2 + x^2} = a + b + \frac{x^2}{2(a+b)}$$

$$SQ - SO' = a + b + \frac{x^2}{2(a+b)} - a$$

$$= b + \frac{x^2}{2(a+b)}$$

$$\delta = OQ - O'Q = b + \frac{x^2}{2b} - b - \frac{x^2}{2(a+b)}$$

$$= \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{a+b}\right)$$

$$= \frac{ax^2}{2b(a+b)}$$

for the displacement Max

$$\delta = (2n+1)\lambda/2 \qquad n = 0, 1, \ldots$$

$$\frac{ax^2}{2b(a+b)} = (2n+1)\lambda/2$$

$$x^2_{max} = \frac{(2n+1)\lambda\, 2b(a+b)}{2a}$$

$$x_{max} = \sqrt{\frac{b(a+b)(2n+1)\lambda}{a}}$$

or

$$x_{max} \propto \sqrt{2n+1}$$

For Minimum

$$\delta = 2n\lambda/2$$

$$\frac{ax^2}{2b(a+b)} = 2n\lambda/2$$

$$x_{min}^2 = \frac{2b(a+b)n\lambda}{a}$$

$$x_{min} = \sqrt{\frac{2b(a+b)n\lambda}{a}}$$

$$x_{min} \propto \sqrt{n\lambda}.$$