ARULMIGU PALANIANDAVAR ARTS COLLEGE FOR WOMEN,

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DEPARTMENT OF MATHEMATICS

LEARNING RSOURCES

SUBJECT: PARTIAL DIFFERENTIAL EQUATIONS

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2/1/2023 Integral Surfaces passing through a Given curve

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1. Find the Integral Surface of the linear Partial differential eqn

$$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$$

which contains the straight line $x+y=0, z=1$

The auxiliary equation

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{(x^2-y^2)z}$$

Taking the first, two, 3 fractions

$$\frac{dx/x}{y^2+z} = \frac{dy/y}{-(x^2+z)} = \frac{dx/x}{y^2+z} + \frac{dy/y}{-(x^2+z)} = \frac{dz/z}{x^2-y^2}.$$

$$\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{y^2+z-x^2-z+x^2-y^2} = \frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{0}$$

$$\Rightarrow \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

Integrating, $\log x + \log y + \log z = \log c_1$

$$xyz = c_1$$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{(x^2-y^2)z}$$

$$\frac{x\,dx}{x^2(y^2+z)} = \frac{y\,dy}{-y^2(x^2+z)} = \frac{dz}{(x^2-y^2)z}.$$

$$\frac{x\,dx+y\,dy}{x^2y^2+x^2z-y^2x^2-y^2z} = \frac{dz}{(x^2-y^2)z}$$

$$\frac{x\,dx+y\,dy}{z(x^2-y^2)} = \frac{dz}{(x^2-y^2)z}$$

$$x\,dx+y\,dy-dz = 0$$

Integrating,

$$\frac{x^2}{2}+\frac{y^2}{2}-z = c_2$$

$$x^2+y^2-2z = c_2$$

Given $x+y=0$, $z=1$

put $x=t$, $y=-t$, $z=1$

$\therefore\ xyz = c_1$ | $x^2+y^2-2z = c_2$

$t(-t)\cdot 1 = c_1$ | $t^2+t^2-2 = c_2$

$-t^2 = c_1$ | $2t^2-2 = c_2$

Eliminating t, we get

$$2c_1+c_2+2 = 0$$

The general surface is

$$x^2+y^2+2xyz-2z+2=0$$

(3) Find the integral of the equation $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ which passes through the curve $xz=a^3, y=0$

Lagrange's auxiliary equation is

$$\frac{dx}{(x-y)y^2}=\frac{dy}{(y-x)x^2}=\frac{dz}{(x^2+y^2)z}.$$

Taking the first two fractions we get

$$\frac{dx}{(x-y)y^2}=\frac{dy}{(y-x)x^2}.$$

$$\frac{x^2dx}{x-y}=\frac{y^2dy}{-(x-y)}$$

Integrating, $\frac{x^3}{3}=\frac{-y^3}{3}+C_1$

$$x^3+y^3=c_1$$

$$\frac{dx-dy}{y^2(x-y)+x^2(x-y)}=\frac{dx-dy}{(x-y)(y^2+x^2)}$$

$$\frac{dx-dy}{(x-y)(y^2+x^2)}=\frac{dz}{z(x^2+y^2)}$$

i.e., $\dfrac{dx - dy}{x - y} = \dfrac{dz}{z}$

Integrating we get

$$\log(x - y) = \log z + \log c_2$$

$$\frac{x - y}{z} = c_2$$

The parametric equation of the given curve is $x = t$, $y = 0$, $z = \dfrac{a^3}{t}$.

Substituting these values, in $x^3 + y^3 = c_1$

and $\dfrac{x - y}{z} = c_2$ we get

$$t^3 = c_1 \quad , \quad \frac{t^2}{a^3} = c_2 \quad \text{or } t^2 = a^3 c_2$$

Eliminating t, we get

$$c_1^2 = a^9 c_2^3$$

$$\text{i.e., } (x^3 + y^3)^2 = a^9 \left(\frac{x - y}{z}\right)^3$$

$$\text{i.e., } z^3 (x^3 + y^3)^2 = a^9 (x - y)^3$$

3) Find the general solution of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$

and also obtain the particular solution which passes through the line $x = 1$, $y = 0$.

Solution

Lagrange's auxiliary equation is

$$\frac{dx}{2xy-1} = \frac{dy}{z-2x^2} = \frac{dz}{2(x-yz)}$$

$$\frac{x\,dx + y\,dy}{2x^2y - x + zy - 2x^2y} = \frac{x\,dx + y\,dy}{-(x-zy)}$$

$$\frac{x\,dx + y\,dy}{-(x-zy)} = \frac{dz}{2(x-yz)}.$$

$$x\,dx + y\,dy = -\frac{dz}{2}$$

Integrating, $\frac{x^2}{2} + \frac{y^2}{2} = -\frac{1}{2}z + c_1$

$$x^2 + y^2 = -z + c_1$$

$$\underline{\underline{x^2 + y^2 + z = c_1}}$$

$$\frac{dx}{2xy-1} = \frac{dy}{z-2x^2} \Rightarrow (z-2x^2)\,dx = (2xy-1)\,dy$$

ie, $(c_1 - 3x^2 - y^2)\,dx = (2xy-1)\,dy$.

$\Rightarrow (c_1 - 3x^2)\,dx - y^2\,dx - 2xy\,dy + dy = 0$

$$(c_1 - 3x^2)\,dx + dy - d(xy^2) = 0$$

Integrating we get

$$c_1 x - x^3 + y - xy^2 = c_2$$

$$(x^2+y^2+z)x - x^3 + y - xy^2 = c_2$$

$$zx + y = c_2 \,//$$

The equation of the given curve is $x=1, y=0$

Putting $x=1, y=0$, in the eqn $zx+y=c_2$, $x^2+y^2+z=c_1$

, we get $\underline{z=c_2}$ and $\underline{1+z=c_1}$

$$c_1 - c_2 = 1$$

Substituting the values for c_1, c_2 in $c_1 - c_2 = 1$

$$x^2+y^2+z - (zx+y) = 1$$

$$x^2+y^2+z-zx-y = 1$$

$$\underline{x^2+y(y-1)+z(1-x)-1=0}$$

1. Find the eqn of the Integral Surface of the PD

$2y(z-3)p + (2x-z)q = y(2x-3)$ which

passes through the circle $z=0$, $x^2+y^2=2x$

The auxiliary system is given by

$$\frac{dx}{2y(z-3)} = \frac{dy}{2x-z} = \frac{dz}{y(2x-3)}$$

Taking the Ist and 3rd fraction

$$\frac{dx}{2y(z-3)} = \frac{dz}{y(2x-3)}$$

$$\frac{dx}{2(z-3)} = \frac{dz}{2x-3}$$

$(2x-3)\,dx = (2z-6)\,dz$

Interating we get

$$x^2 - 3x = z^2 - 6z + C_1$$

$$x^2 - 3x - z^2 + 6z = C_1$$

$$\frac{y\,dy - dz}{2xy - yz - 2yx + 3y} = \frac{y\,dy - dz}{-yz + 3y} = \frac{y\,dy - dz}{y(3-z)}.$$

$$\frac{dx}{2y(z-3)} = \frac{y\,dy - dz}{y(3-z)}$$

$$\frac{dx}{2y(z-3)} = \frac{y\,dy - dz}{y(3-z)}$$

$$\frac{dx}{2} = \frac{y\,dy - dz}{-1}$$

$$dx + 2y\,dy - 2dz = 0$$

Inlerating $\underline{x + y^2 - 2z = C_2}$

Now the given curve is $x^2+y^2 = 2x,\ z=0$

This eqn can be written as $(x-1)^2 + (y-0)^2 = 1$

with centre $(1,0)$ and radius 1

The corresponding parameltic eqn is

$$x - 1 = \cos\theta, \quad y - 0 = \sin\theta,\ z = 0$$

$$x = 1 + \cos\theta, \quad y = \sin\theta,\ z = 0$$

$$1 + \cos\theta + \sin^2\theta - 2(0) = C_2$$

$$1 + \cos\theta + 1 - \cos^2\theta = C_2$$

$$2 + \cos\theta - \cos^2\theta = C_2$$

$$(1+\cos\theta)^2 - 3(1+\cos\theta) - 0^2 + 6(0) = C_1$$

$$1 + 2\cos\theta + \cos^2\theta - 3 - 3\cos\theta = C_1$$

$$\cos^2\theta - 2 - \cos\theta = C_1$$

$$-(2 + \cos\theta - \cos^2\theta) = C_1$$

$$C_1 = -C_2$$

$$C_1 + C_2 = 0$$

$$x^2 - 3x - z^2 + 6z + x + y^2 - 2z = 0$$

$$x^2 + y^2 - z^2 - 2x + 4z = 0$$

Solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

Lagrange's auxiliary equation is

$$\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} \quad -(1)$$

Taking $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multipliers we get

$$\frac{\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}}{x(y-z)+y(z-x)+z(x-y)} = \frac{\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}}{0}$$

Considering this with any fraction in eqn (1) we get

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

Integrating, $\log x + \log y + \log z = \log c_1$

$$\underline{\underline{xyz = c_1}}$$

Again taking $\frac{1}{x^2}, \frac{1}{y^2}$ & $\frac{1}{z^2}$ as multipliers

$$\frac{\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2}}{y-z+z-x+x-y} = \frac{\frac{dx}{x^2}+\frac{dy}{y^2}+\frac{dz}{z^2}}{0}$$

ie, $\frac{dx}{x^2} + \frac{dy}{y^2} + \frac{dz}{z^2} = 0$.

Integrating $-\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = c_2$.

ie, $\underline{\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = c_2}$

Hence the general soln is

$$\phi\left(xyz, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=0$$

2) Solve $(y^2+z^2)p - xyq + xz = 0$

Lagrange's auxiliary eqn is

$$\frac{dx}{y^2+z^2} = \frac{dy}{-xy} = \frac{dz}{-xz} \quad \text{— (A)}$$

Taking the last two fractions we get

$$\frac{dy}{-xy} = \frac{dz}{-xz}$$

ie, $\dfrac{dy}{y} = \dfrac{dz}{z}$.

Integrating, $\log y = \log z + \log c_1$

$$\log y - \log z = \log c_1$$

ie, $\dfrac{y}{z} = c_1$

Taking x, y, z as multipliers, each fraction in eq(1) is ~~aeqgt~~ equal to

$$\frac{x\,dx + y\,dy + z\,dz}{xy^2 + xz^2 - xy^2 - xz^2} = \frac{x\,dx + y\,dy + z\,dz}{0}$$

ie, $x\,dx + y\,dy + z\,dz = 0$

Integrating, $\dfrac{x^2}{2} + \dfrac{y^2}{2} + \dfrac{z^2}{2} = \dfrac{c_2}{2}$

$$x^2 + y^2 + z^2 = c_2$$

Hence the general solution is

$$\phi\left(\frac{y}{z},\ x^2+y^2+z^2\right)=0$$

3) Solve $p - 2q = 3x^2 \sin(y+2x)$

Lagrange's auxiliary equation is

$$\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2 \sin(y+2x)} \quad -(I)$$

Taking the first two fractions we have.

$$\frac{dx}{1} = \frac{dy}{-2} \Rightarrow 2dx + dy = 0$$

i.e. $2x + y = c_1 \quad -(II)$

Taking the first and third fraction ~~and~~ and using eqn (II)

$$\frac{dx}{1} = \frac{dz}{3x^2 \sin(y+2x)}$$

$$\frac{dx}{1} = \frac{dz}{3x^2 \sin c_1}$$

$$3(\sin c_1)\, x^2\, dx = dz$$

$$x^3 \sin c_1 - z = c_2$$

$$x^3 \sin(y+2x) - z = c_2$$

Hence the general soln is

$$\phi\left(2x+y,\ x^3 \sin(2x+y)-z\right)=0$$

4) Solve $\dfrac{dx}{(x-y)y^2} = \dfrac{dy}{(y-x)x^2} = \dfrac{dz}{(x^2+y^2)z}$

Taking the first two fraction we get

$$\frac{dx}{(x-y)y^2} = \frac{dy}{-(x-y)x^2} \Rightarrow x^2dx+y^2dy=0$$

$\Rightarrow$ Integrating we get

$$\frac{x^3}{3}+\frac{y^3}{3}=C_1$$

ie, $x^3+y^3=C_1$

$$\frac{dx-dy}{y^2(x-y)+x^2(x-y)} = \frac{dx-dy}{(x-y)(y^2+x^2)}$$

$$\frac{dx-dy}{(x-y)(x^2+y^2)} = \frac{dz}{(x^2+y^2)z}$$

ie, $\dfrac{dx-dy}{x-y} = \dfrac{dz}{z}$

Integrating $\log(x-y) = \log z + \log c_2$

$$\frac{x-y}{z} = c_2$$

Sln is $\phi\left(x^3+y^3,\ \dfrac{x-y}{z}\right)=0$ //

Sravellu.T.S

Eliminate the ordinary constants for the equation $x^2+y^2+(z-c)^2=a^2$ where the constant a and c are arbitrary

Sln: Given equation $x^2+y^2+(z-c)^2=a^2$ — ①

Diff. eqn ① partially w.r.to x

$$2x+2(z-c)\cdot\frac{\partial z}{\partial x}=0$$

$$2x+2(z-c)\cdot p=0$$

$$x+(z-c)p=0$$

$$\Rightarrow (z-c)p=-x$$

$$z-c=-x/p \quad \text{— ②}$$

Diff. eqn ② partially w.r.to y

$$2y+2(z-c)\frac{\partial z}{\partial y}=0$$

$$2y+2(z-c)\cdot q=0$$

$$(z-c)q=-y$$

$$z-c=\frac{-y}{q} \quad \text{— ②}$$

Equating equations ②, ③

$$\frac{-x}{p}=-\frac{y}{q}$$

$$xq=yp \Rightarrow xq-yp=0$$

2) Eliminate the constant from the given equation

$x^2+y^2=(z-c)^2\tan^2\alpha$ where the constant c and α

are arbitrary

Given eqn is $x^2+y^2=(z-c)^2\tan^2\alpha$ — ①

Diff. eqn ① partially w.r to x

$$2x = 2(z-c)\frac{\partial z}{\partial x}\tan^2\alpha$$

$$x = (z-c)\,p\tan^2\alpha$$

$$\frac{x}{p} = (z-c)\tan^2\alpha \quad — ②$$

Diff. eqn ① partially w.r to y

$$2y = 2(z-c)\frac{\partial z}{\partial y}\tan^2\alpha$$

$$\text{i.e., } y = (z-c)\,q\tan^2\alpha$$

$$\frac{y}{q} = (z-c)\tan^2\alpha \quad — ③$$

Equating ②, ③ $\frac{x}{p}=\frac{y}{q}$

$$xq = yp \Rightarrow \underline{xq-yp=0}$$

3. Eliminate the constant from the equation

$(x-a)^2+(y-b)^2+z^2=1$ where a, b are

arbitrary constants

Given equation is $(x-a)^2+(y-b)^2+z^2=1$ — ①

Diff. ① partially w.r to x

$$2(x-a)+2z\cdot\frac{\partial z}{\partial x}=0$$

$$2(x-a) = -2zp$$

$$x-a = -zp \Rightarrow \frac{x-a}{p} = z \quad — (2)$$

Diff (1) partially w.r.to y

$$2(y-b) + 2z \cdot \frac{\partial z}{\partial y} = 0$$

$$y-b = -zq$$

$$\frac{y-b}{q} = -z \quad — (3)$$

Equating (2), (3) $\Rightarrow \frac{x-a}{p} = \frac{y-b}{q}$

$$q(x-a) = p(y-b)$$

$$q(x-a) - p(y-b) = 0$$

$$\left.\begin{aligned} (2) &\Rightarrow -zp = x-a \\ (3) &\Rightarrow -zq = y-b \end{aligned}\right] \text{ in } (1)$$

$$(-zp)^2 + (-zq)^2 + z^2 = 1$$

$$z^2p^2 + z^2q^2 + z^2 = 1$$

$$\Rightarrow z^2(p^2+q^2+1) = 1$$

$$\text{ie } z^2(p^2+q^2+1) - 1 = 0$$

4. Eliminate the constant term from the eqn $z = (x+a)(y+b)$ where a, b are constants

Given eqn is $z = (x+a)(y+b)$ — ①

Diff with ① partially w.r to x

$$\frac{\partial z}{\partial x} = y+b \Rightarrow p = y+v \text{ — ②}$$

$$\frac{\partial z}{\partial y} = x+a \Rightarrow q = x+a \text{ — ③}$$

Sub ② & ③ in ① we get

$$z = pq$$

i, $z - pq = 0$

5. Eliminate the constant from the equation

$$2z = (ax+y)^2 + b.$$

Given eqn is $2z = (ax+y)^2 + b$ — ①

Diff. ① partially w.r to x

$$2 \cdot \frac{\partial z}{\partial x} = 2(ax+y) \cdot a$$

$$p = (ax+y) a$$

$$\frac{p}{a} = ax+y \text{ — ②}$$

Diff ① partially with resp. to y

$$2 \cdot \frac{\partial z}{\partial y} = 2(ax+y) \text{ — ②}$$

$$\frac{\partial z}{\partial y} = ax+y$$

$$q = ax+y \text{ — ③}$$

Equating ②, ③ we get

$$\frac{p}{a} = q \Rightarrow \frac{p}{q} = a.$$

$$p = qa.$$

$$p - aq = 0 \Rightarrow q = \left(\frac{p}{q}\right) x + y.$$

$$q = \frac{px + qy}{q}$$

ie, $q^2 = px + qy$

6. Eliminate the constant from the equation

$$ax^2 + by^2 + z^2 = 1.$$

Given eqn is $ax^2 + by^2 + z^2 = 1$ — (1)

Diff (1) partially w.resp. to x.

$$2ax + \cancel{0} + 2z \cdot \frac{\partial z}{\partial x} = 0$$

$$2ax + 2z \cdot \frac{\partial z}{\partial x} = 0$$

$$2z \cdot p = -2ax.$$

$$zp = -ax \Rightarrow a = \frac{-zp}{x}$$

Diff. (1) partially w.r.to y

$$0 + 2by + 2z \cdot \frac{\partial z}{\partial y} = 0$$

$$2 \cdot by = -2zp \cdot \frac{\partial z}{\partial y}$$

$$2by = -2 \cdot qz$$

$$by = -qz \Rightarrow b = \frac{-zq}{y} \quad \text{— (3)}$$

Equating (2), (3) we get

$$\frac{ax}{p} = \frac{by}{q} \quad a, \left(-\frac{zp}{x}\right) x^2 + \left(-\frac{zq}{y}\right) y^2 + z^2 = 1$$

$axq = byq$, $axq - byp = 0$

$$-zpx - zqy + z^2 = 1$$

$$-zpx - zqy = -z^2 + 1$$

$$z(px + qx) = z^2 - 1$$

2. b) Eliminate the arbitrary function f from the eqs

b) $z = x + y + f(xy)$

Given eqn $z = x + y + f(xy)$ — (1)

diff. (1) partially w.r.to x

$$\frac{\partial z}{\partial x} = 1 + 0 + f'(xy) \cdot y$$

$$\frac{p-1}{y} = f'(xy)$$

$$\frac{p-1}{y} = f'(xy) \quad — (2)$$

diff. (1) partially w.r.to y

$$\frac{\partial z}{\partial y} = 1 + 0 + f'(xy) \cdot x \Rightarrow \frac{q-1}{x} = f'(xy) \quad (3)$$

Equating (2), (3) $\Rightarrow \frac{p-1}{y} = \frac{q-1}{x}$

$$x(p-1) = y(q-1)$$

$$\underline{px - x = qy - y}$$

$$-x + y - yq + xp$$

$$\underline{px - qy - x + y = 0}$$

3. Eliminate the arbitrary function from the equation

$$z = f\left(\frac{xy}{z}\right).$$

Given that $z = f\left(\frac{xy}{z}\right)$ — ①

Diff ① partially w.r to x

$$\frac{\partial z}{\partial x} = f^{-1}\left(\frac{xy}{z}\right)\left[\frac{zy - xy \cdot \partial z/\partial x}{z^2}\right]$$

$$p = f'\left(\frac{xy}{z}\right)\left[\frac{zy - xyp}{z^2}\right]$$

$$\frac{pz^2}{zy - xyp} = f^{-1}(xy/z) \quad \text{—②}$$

Diff ① partially w resp. to y

$$\frac{\partial z}{\partial y} = f'\left(\frac{xy}{z}\right)\left[\frac{zx - xy \cdot \partial z/\partial y}{z^2}\right]$$

$$q = f'(xy/z)\left[\frac{zx - xyq}{z^2}\right]$$

$$\frac{qz^2}{zx - xyq} = f'\left(\frac{xy}{z}\right) \quad \text{—③}$$

Equating ②, ③ we get

$$\frac{pz^2}{zy - xyp} = \frac{qz^2}{zx - xyq}$$

$$p(zx - xyq) = q(zy - xyp)$$

$$pzx - pxyq = qzy - xyqp$$

$$xpz - pxyq - qzy + xypq = 0$$

$$xpz - yqz = 0$$

$$z(xp - yq) = 0$$

$$\underline{xp - yq = 0}$$

9) Eliminate the arbitrary function from the eqn

$$z = f(x-y)$$

Given eqn $z = f(x-y)$ — (1)

Diff (1) partially w. resp to x

$$\frac{\partial z}{\partial x} = f'(x-y)$$

$$p = f'(x-y) \quad \text{— (2)}$$

Diff (1) partially w.resp. to y

$$\frac{\partial z}{\partial y} = f'(x-y) \times (-1)$$

$$-q = f'(x-y) \quad \text{— (3)}$$

Equating (2), (3) →

$$-q = p$$

ie, $\underline{p+q = 0}$

Simultaneous

Find the integral Surface of the linear partial diff. eqn

$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0$, $z=1$

The given eqn is $x(y^2+z) - y(x^2+z)q = z(x^2-y^2)$

Lagrange's eqn is $Pp + Qq = R$

$P = x(y^2+z)$, $Q = -y(x^2+z)$, $R = z(x^2-y^2)$

The subsidary eqn is

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$$

choose $(x, y, -1)$ as Lagrange's multipliers

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2-y^2)} \Rightarrow$$

$$\frac{x\,dx + y\,dy - dz}{x^2y^2 + zx^2 - x^2y^2 - zy^2 - zx^2 + zy^2}$$

$$\Rightarrow \frac{x\,dx + y\,dy - dz}{0} \Rightarrow x\,dx + y\,dy - dz = 0$$

Integrating on b.s

$$\int x\,dx + \int y\,dy - \int dz = 0 + c$$

$$\frac{x^2}{2} + \frac{y^2}{2} - z = 0 + c$$

$$\frac{x^2+y^2-2z}{z} = c \Rightarrow x^2+y^2-2z = 2c$$

$$\Rightarrow x^2+y^2-2z = c_1$$

Choose $\left(1/x,\ 1/y\ \ldots\right)$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{z(x^2+y^2)}$$

$$= \frac{1/x\, dx + 1/y\, dy + 1/z\, dz}{y^2+z-x^2-z+x^2-y^2}$$

$$\Rightarrow \frac{\frac{1}{x}dx + \frac{1}{y}\cdot dy + \frac{1}{z}dz}{0}$$

$$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

Integrating, $\int \frac{dx}{x} + \int \frac{dy}{y} + \int \frac{dz}{z} = 0$

$$\Rightarrow \log x + \log y + \log z = \log c$$

$$\Rightarrow \log(xyz) = \log c.$$

$$xyz = c_2 \quad \text{— (2)}$$

$$x^2+y^2-2z = c_1$$

$$xyz = c_2$$

$x+y = 0,\ z=1$ — (4)

Assume that $x = t$

Sub. $x = t$ in eqn (4) we get

$t + y = 0, \; z = 1$

$y = -t, \; z = 1$

Sub. $x = t, \quad y = -t, \quad z = 1$ in (3)

$t^2 + t^2 - 2(1) = c_1$

$2t^2 - 2 = c_1 \quad -5$

Solve eqn (3) and (6) $\Rightarrow$ $2t^2 - 2 = c_1$

$6 \times 2 \Rightarrow -2t^2 = 2c_2$

$-2 = c_1 + 2c_2$

$c_1 + 2c_2 + 2 = 0$

Sub. eqn (3) in 7

$x^2 + y^2 - 2z + 2(xyz) + 2 = 0$

$x^2 + y^2 + 2xyz - 2z + 2 = 0$

which is the required integral surface.