MATRICES

INTRODUCTION

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Matrices - Introduction

Matrix algebra has at least two advantages:

- **Reduces complicated systems of equations to simple expressions**
- **Adaptable to systematic method of mathematical treatment and well suited to computers**

Definition:

A matrix is a set or group of numbers arranged in a square or rectangular array enclosed by two brackets

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 4 & 2 \\ -3 & 0 \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Matrices - Introduction

Properties:

- A specified number of rows and a specified number of columns
- Two numbers (rows x columns) describe the dimensions or size of the matrix.

Examples:

3x3 matrix

2x4 matrix

1x2 matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ 3 & 3 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 3 & -3 \\ 0 & 0 & 3 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 \end{bmatrix}$$

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A matrix is denoted by a bold capital letter and the elements within the matrix are denoted by lower case letters

e.g. matrix [A] with elements a_{ij}

A_{mxn}=

${}_mA^n$

$$\begin{bmatrix} a_{11} & a_{12}\cdots & a_{ij} & a_{in} \\ a_{21} & a_{22}\cdots & a_{ij} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{ij} & a_{mn} \end{bmatrix}$$

i goes from 1 to m

j goes from 1 to n

Matrices - Introduction

TYPES OF MATRICES

1. Column matrix or vector:

The number of rows may be any integer but the number of columns is always 1

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

Matrices - Introduction

TYPES OF MATRICES

2. Row matrix or vector

Any number of columns but only one row

$$\begin{bmatrix} 1 & 1 & 6 \end{bmatrix} \qquad \begin{bmatrix} 0 & 3 & 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1n} \end{bmatrix}$$

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TYPES OF MATRICES

3. Rectangular matrix

Contains more than one element and number of rows is not equal to the number of columns

$$\begin{bmatrix} 1 & 1 \\ 3 & 7 \\ 7 & -7 \\ 7 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

$$m \neq n$$

Matrices - Introduction

TYPES OF MATRICES

4. Square matrix

The number of rows is equal to the number of columns

(a square matrix A has an order of m)

m x m

$$\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

The principal or main diagonal of a square matrix is composed of all elements a_{ij} for which $i=j$

Matrices - Introduction

TYPES OF MATRICES

5. Diagonal matrix

A square matrix where all the elements are zero except those on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$

$a_{ij} \neq 0$ for some or all $i = j$

Matrices - Introduction

TYPES OF MATRICES

6. Unit or Identity matrix - I

A diagonal matrix with ones on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} a_{ij} & 0 \\ 0 & a_{ij} \end{bmatrix}$$

i.e. a_{ij}=0 for all $i = j$

a_{ij}= 1 for some or all $i = j$

Matrices - Introduction

TYPES OF MATRICES

7. Null (zero) matrix - 0

All elements in the matrix are zero

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a_{ij} = 0$$ **For all *i,j***

Matrices - Introduction

TYPES OF MATRICES

8. Triangular matrix

A square matrix whose elements above or below the main diagonal are all zero

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 8 & 9 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

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TYPES OF MATRICES

8a. Upper triangular matrix

A square matrix whose elements below the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & a_{ij} & a_{ij} \\ 0 & a_{ij} & a_{ij} \\ 0 & 0 & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 1 & 8 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 7 & 4 & 4 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i > j$

TYPES OF MATRICES

8b. Lower triangular matrix

A square matrix whose elements above the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ a_{ij} & a_{ij} & 0 \\ a_{ij} & a_{ij} & a_{ij} \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i < j$

TYPES OF MATRICES

9. Scalar matrix

A diagonal matrix whose main diagonal elements are equal to the same scalar

A scalar is defined as a single number or constant

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ 0 & a_{ij} & 0 \\ 0 & 0 & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i = j$

$a_{ij} = a$ for all $i = j$

Adjoint Matrix

Adjoint of a Matrix

DEFINITION

If A is any $n \times n$ matrix and C_{ij} is the cofactor of a_{ij}, then the matrix

$$\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix}$$

is called the ***matrix of cofactors from A***. The transpose of this matrix is called the ***adjoint of A*** and is denoted by $\text{adj}(A)$.

ADJOINT OF A MATRIX

Let

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

The cofactors of A are

$$\begin{array}{lll} C_{11} = 12 & C_{12} = 6 & C_{13} = -16 \\ C_{21} = 4 & C_{22} = 2 & C_{23} = 16 \\ C_{31} = 12 & C_{32} = -10 & C_{33} = 16 \end{array}$$

so the matrix of cofactors is

$$\begin{bmatrix} 12 & 6 & -16 \\ 4 & 2 & 16 \\ 12 & -10 & 16 \end{bmatrix}$$

and the adjoint of A is

$$\operatorname{adj}(A) = \begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$$

Co factor Matrix

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$\underline{M}_{11}$: remove row 1, col 1

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix} \quad \underline{M}_{11} = \begin{pmatrix} 2 & 3 \\ 7 & 0 \end{pmatrix}$$

Determinants

- To define det(A) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$\underline{M}_{12}$: remove row 1, col 2

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$$\underline{M}_{12} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$$

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$\underline{M}_{13}$: remove row 1, col 3

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$$\underline{M}_{13} = \begin{pmatrix} -1 & 2 \\ 2 & 7 \end{pmatrix}$$

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$\underline{M}_{21}$: remove row 2, col 1

$$\underline{M}_{21} = \begin{pmatrix} 1 & -2 \\ 7 & 0 \end{pmatrix}$$

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$\underline{M}_{22}$: remove row 2, col 2

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$$\underline{M}_{22} = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$$

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$\underline{M}_{23}$: remove row 2, col 3

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$$\underline{M}_{23} = \begin{pmatrix} 1 & 1 \\ 2 & 7 \end{pmatrix}$$

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$\underline{M}_{31}$: remove row 3, col 1

$$\underline{M}_{31} = \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}$$

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$\underline{M}_{32}$: remove row 3, col 2

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$$\underline{M}_{32} = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$$

Determinants

- To define det($\underline{A}$) for larger matrices, we will need the definition of a **minor** $\underline{M}_{ij}$
- The minor $\underline{M}_{ij}$ of a matrix $\underline{A}$ is the matrix formed by removing the ^{i}th row and the ^{j}th column of $\underline{A}$

$$\underline{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 3 \\ 2 & 7 & 0 \end{pmatrix}$$

$\underline{M}_{33}$: remove row 3, col 3

$$\underline{M}_{33} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

Co – Factor Matrix

Find co factor matrix $\begin{bmatrix} 3 & -7 & 5 \\ 2 & 1 & 0 \\ -9 & 2 & 4 \end{bmatrix}$

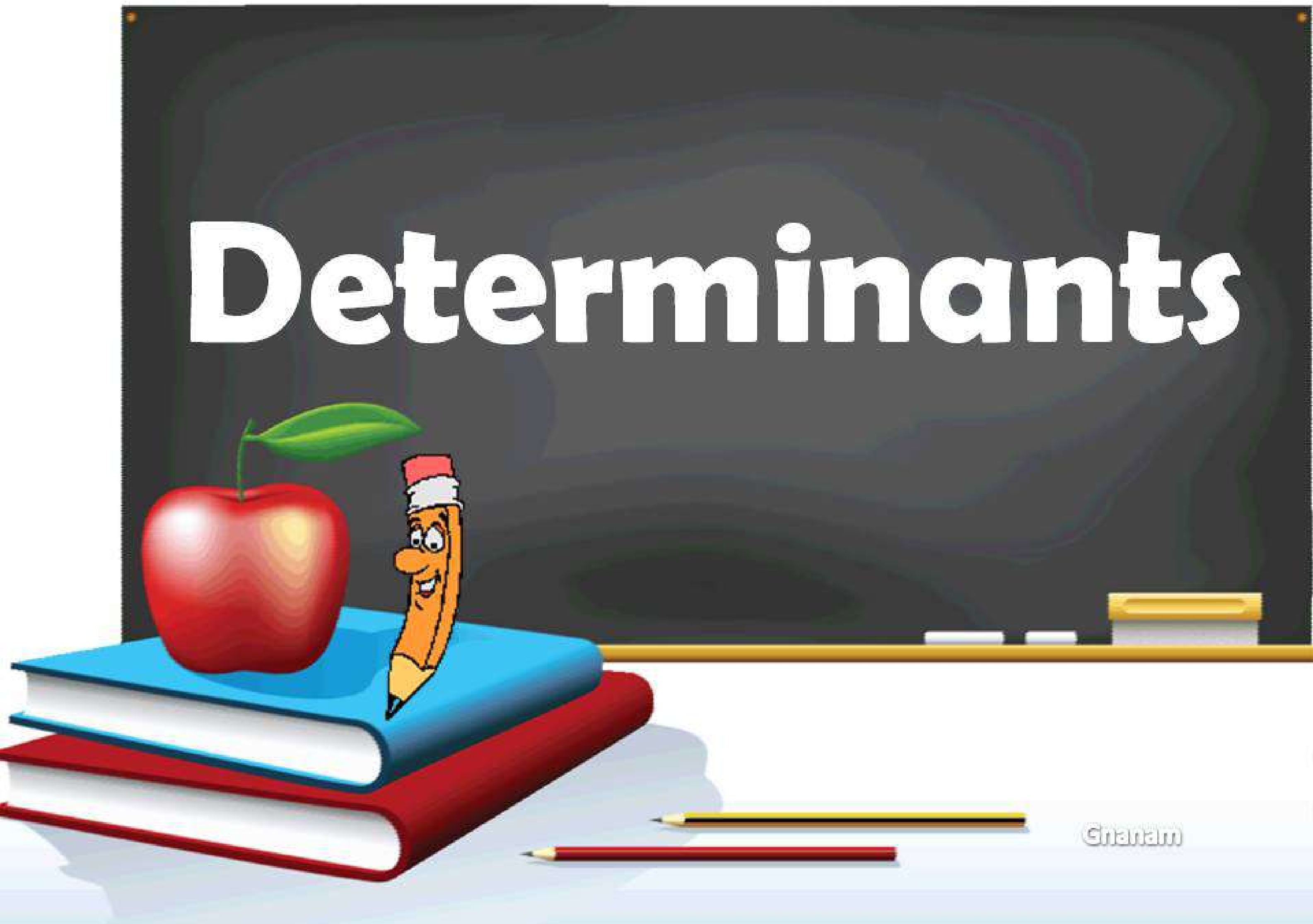
Co-Factor of 3 is $A_{11} = (-1)^{1+1} . M_{11} = (-1)^2 \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} = 1 |1.4 - 2.0| = 1 |4 - 0|$

$= 1. 4 = 4$

Co-Factor of -7 is $A_{12} = (-1)^{1+2} . M_{12} = (-1)^3 \begin{bmatrix} 2 & 0 \\ -9 & 4 \end{bmatrix} = -1 |2.4 - (-9.0)| = -1 |8 + 0|$

$= -1. 8 = -8$

Same procedure apply for all elements


Determinants
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Determinants

- Given a **square matrix** A its **determinant** is a real number associated with the matrix.
- The determinant of A is written:

$$\det(\underline{A}) \quad \text{or} \quad |\underline{A}|$$

- For a 2x2 matrix, the definition is

$$\text{det.}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- For larger matrices the definition is more complicated

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DETERMINANT

Every square matrix has associated with it a scalar called its determinant.

Given a matrix **A**, we use **det(A)** or **|A|** to designate its determinant.

We can also designate the determinant of matrix **A** by replacing the brackets by vertical straight lines. For example,

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \quad \det(A) = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}$$

Definition 1: The determinant of a 1×1 matrix [a] is the scalar a.

Definition 2: The determinant of a 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is the scalar ad-bc.

For higher order matrices, we will use a recursive procedure to compute determinants.

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Example

Evaluate the determinant: $\begin{vmatrix} 4 & -3 \\ 2 & 5 \end{vmatrix}$

Solution: $\begin{vmatrix} 4 & -3 \\ 2 & 5 \end{vmatrix} = 4 \times 5 - 2 \times (-3) = 20 + 6 = 26$

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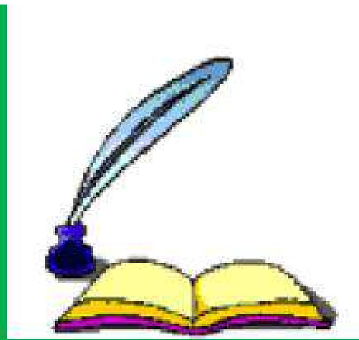

Solution

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a square matrix of order 3, then

$$|A| = \begin{vmatrix} \overset{+}{a_{11}} & \overset{-}{a_{12}} & \overset{+}{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

[Expanding along first row]

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$$= (a_{11}a_{22}a_{33} + a_{12}a_{31}a_{23} + a_{13}a_{21}a_{32}) - (a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33} + a_{13}a_{31}a_{22})$$

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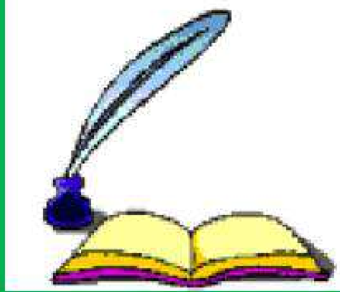
Example

Evaluate the determinant : $\begin{vmatrix} 2 & 3 & -5 \\ 7 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix}$



Solution :

$$\begin{matrix} + & - & + \end{matrix}$$

$$\begin{vmatrix} 2 & 3 & -5 \\ 7 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} + (-5)\begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix}$$

[Expanding along first row]

$$= 2\,|1x1 - (4x -2)| - 3|\,7x1) - (-3x-2) - 5|\,(7x4\,) - (-3x1$$

$$= 2(1 + 8) - 3(7 - 6) - 5(28 + 3)$$

$$= 18 - 3 - 155$$

$$= -140$$

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Inverse of Matrix

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INVERSE OF MATRIX

2 X 2

3 X 3

Formula :

$$A^{-1} = \frac{1}{|A|} \times adj A$$

Ex 2 X 2

$$A = \begin{bmatrix} 5 & 4 \\ 5 & 6 \end{bmatrix}. \ Find \quad A^{-1}.$$

1. Determinant A

$$|A| = 5(6) - 5(4)$$

$$|A| = 30 - 20$$

$$|A| = 10$$

OBJECTIVES :
-Know characteristics of matrices
-Apply basic operations on matrices
-Know the inverse matrices (up to 3X3)
-Solve simultaneous linear equations up to 3 variables

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$$A = \begin{bmatrix} 5 & 4 \\ 5 & 6 \end{bmatrix}$$

2. Minor A

$$M = \begin{bmatrix} 5 & 4 \\ 5 & 6 \end{bmatrix}$$

What you can see??

$m_{11} = 6$ $m_{12} = 5$

$m_{21} = 4$ $m_{22} = 5$

$$Minor = \begin{bmatrix} 6 & 5 \\ 4 & 5 \end{bmatrix}$$

OBJECTIVES :

- -Know characteristics of matrices
- -Apply basic operations on matrices
- -Know the inverse matrices (up to 3X3)
- -Solve simultaneous linear equations up to 3 variables

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3. Cofactor A

$$Cofactor = \begin{bmatrix} 6 & -5 \\ -4 & 5 \end{bmatrix}$$

4. Adjoint A

$$Adj\ A = \begin{bmatrix} 6 & -4 \\ -5 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times adjA$$

5. Inverse A

$$A^{-1} = \frac{1}{10} \times \begin{bmatrix} 6 & -4 \\ -5 & 5 \end{bmatrix} = \frac{1}{10} \times \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

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INVERSE OF MATRIX 3 X 3

Formula :

$$A^{-1} = \frac{1}{|A|} \times adjA$$

Example :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 5 & -3 & 2 \\ 7 & 1 & 3 \end{bmatrix} \quad Find \quad A^{-1}$$

1. <u>Determinant A</u>

$$|A| = 3$$

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2. Cofactor

$$K = \begin{bmatrix} (+)\,{-11} & (-)\,1 & (+)\,26 \\ (-)\,2 & (+)\,{-1} & (-)\,{-5} \\ (+)\,5 & (-)\,{-1} & (+)\,{-11} \end{bmatrix}$$

$$K = \begin{bmatrix} -11 & -1 & 26 \\ -2 & -1 & 5 \\ 5 & 1 & -11 \end{bmatrix}$$

3. Adjoint

$$adj\ A = \begin{bmatrix} -11 & -2 & 5 \\ -1 & -1 & 1 \\ 26 & 5 & -11 \end{bmatrix}$$

4. Inverse A

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -11 & -2 & 5 \\ -1 & -1 & 1 \\ 26 & 5 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -\frac{11}{3} & -\frac{2}{3} & \frac{5}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{26}{3} & \frac{5}{3} & -\frac{11}{3} \end{bmatrix}$$

Properties of Determinants

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Properties of Determinants

1. The value of a determinant remains unchanged, if its rows and columns are interchanged.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad \text{i.e. } |A| = |A'|$$

2. If any two rows (or columns) of a determinant are interchanged, then the value of the determinant is changed by minus sign.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = - \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad [\text{Applying } R_2 \leftrightarrow R_1]$$

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Properties

3. If all the elements of a row (or column) is multiplied by a non-zero number k, then the value of the new determinant is k times the value of the original determinant.

$$\begin{vmatrix} ka_1 & kb_1 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = k \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

which also implies

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \frac{1}{m} \begin{vmatrix} ma_1 & mb_1 & mc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Properties

4. If each element of any row (or column) consists of two or more terms, then the determinant can be expressed as the sum of two or more determinants.

$$\begin{vmatrix} a_1 + x \\ a_2 + y \\ a_3 + z \end{vmatrix} \quad \begin{vmatrix} \\ \\ \\ \end{vmatrix} \quad \begin{vmatrix} \\ \\ \\ \end{vmatrix}$$

5. The value of a determinant is unchanged, if any row (or column) is multiplied by a number and then added to any other row (or column).

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + mb_1 - nc_1 & b_1 & c_1 \\ a_2 + mb_2 - nc_2 & b_2 & c_2 \\ a_3 + mb_3 - nc_3 & b_3 & c_3 \end{vmatrix} \quad [\text{Applying } C_1 \rightarrow C_1 + mC_2 - nC_3]$$

Properties

6. If any two rows (or columns) of a determinant are identical, then its value is zero.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$$

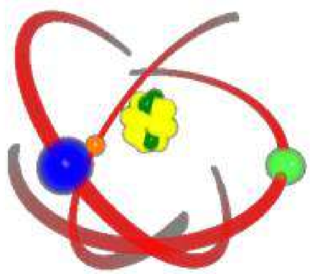
7. If each element of a row (or column) of a determinant is zero, then its value is zero.

$$\begin{vmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Properties

(8) Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ be a diagonal matrix, then

$$|A| = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc$$



Thank you

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