MECHANICS AND PROPERTIES OF MATTER

Learning Material

Prepared by

DR.R.VALLIAMMAL

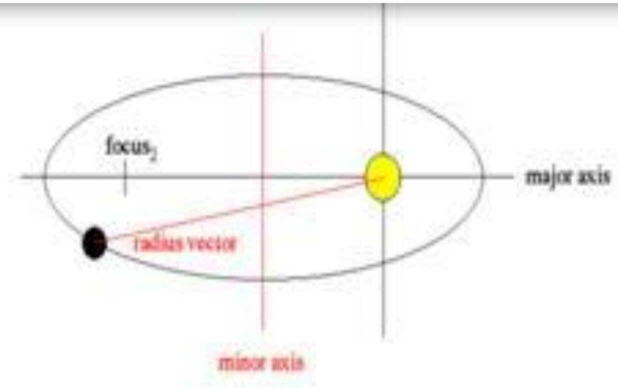
ASSOCIATE PROFESSOR OF PHYSICS

APAC W, PALANI

Kepler’s Laws of Planetary motion

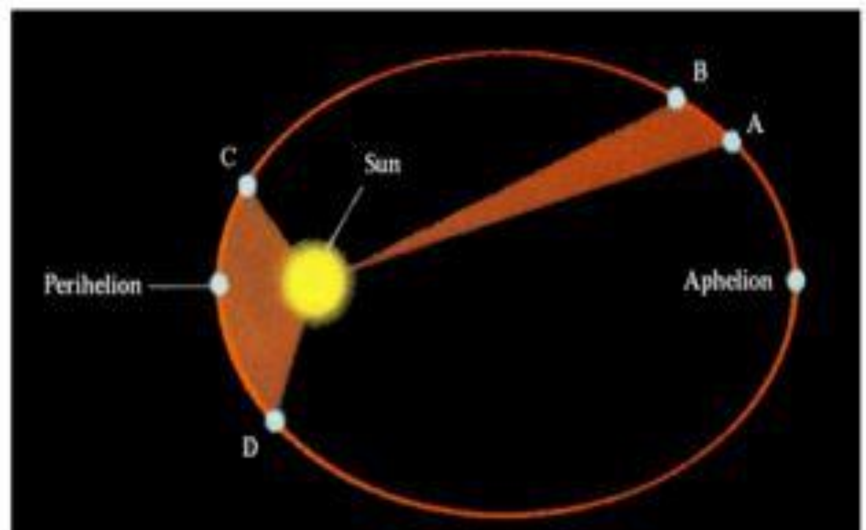
First Law

Each planet moves in an elliptical orbit with it’s star (Sun) at one focus



Second Law

(law of equal areas): an orbiting object will take the same amount of time to travel between points A & B as it takes to travel between points C & D

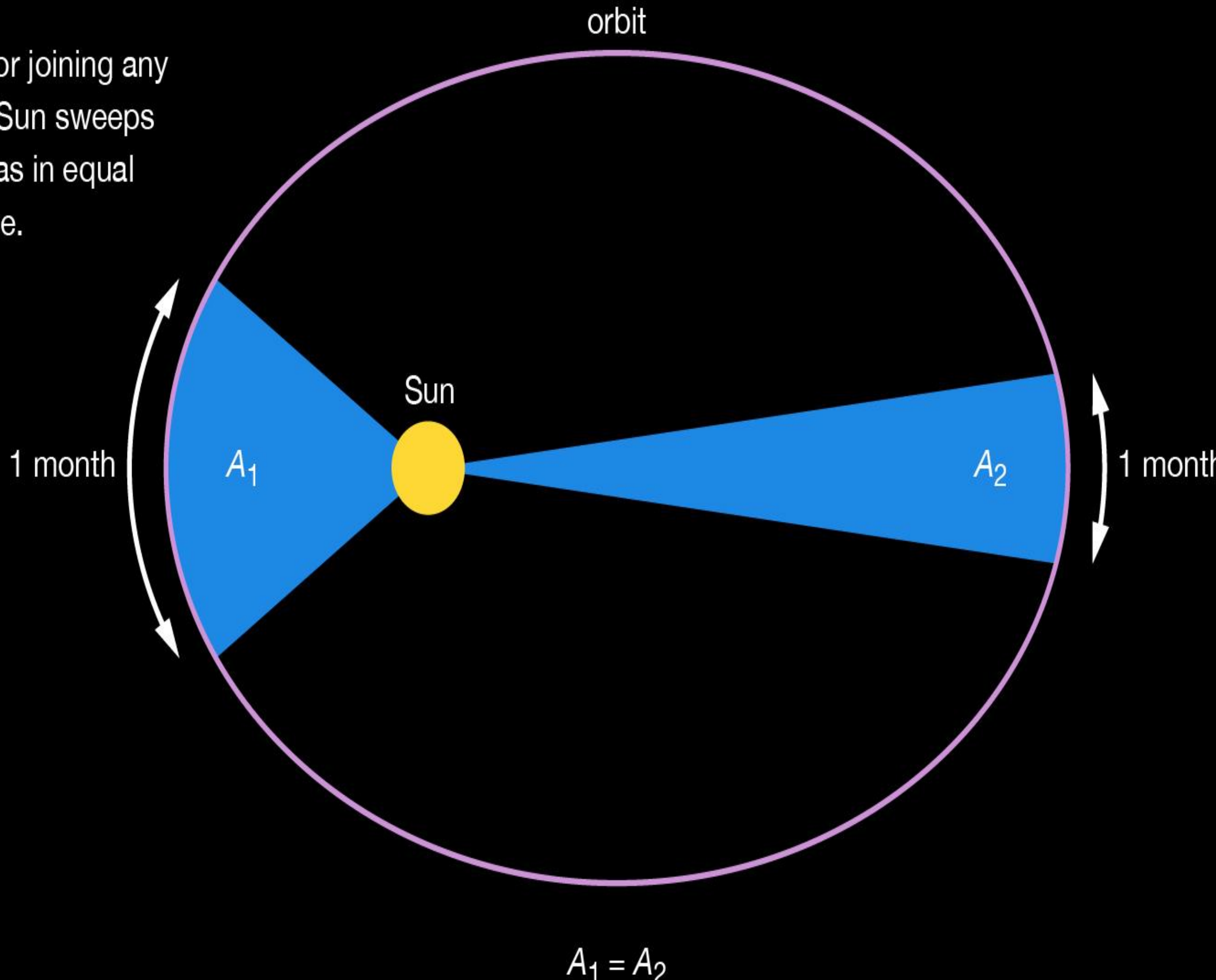


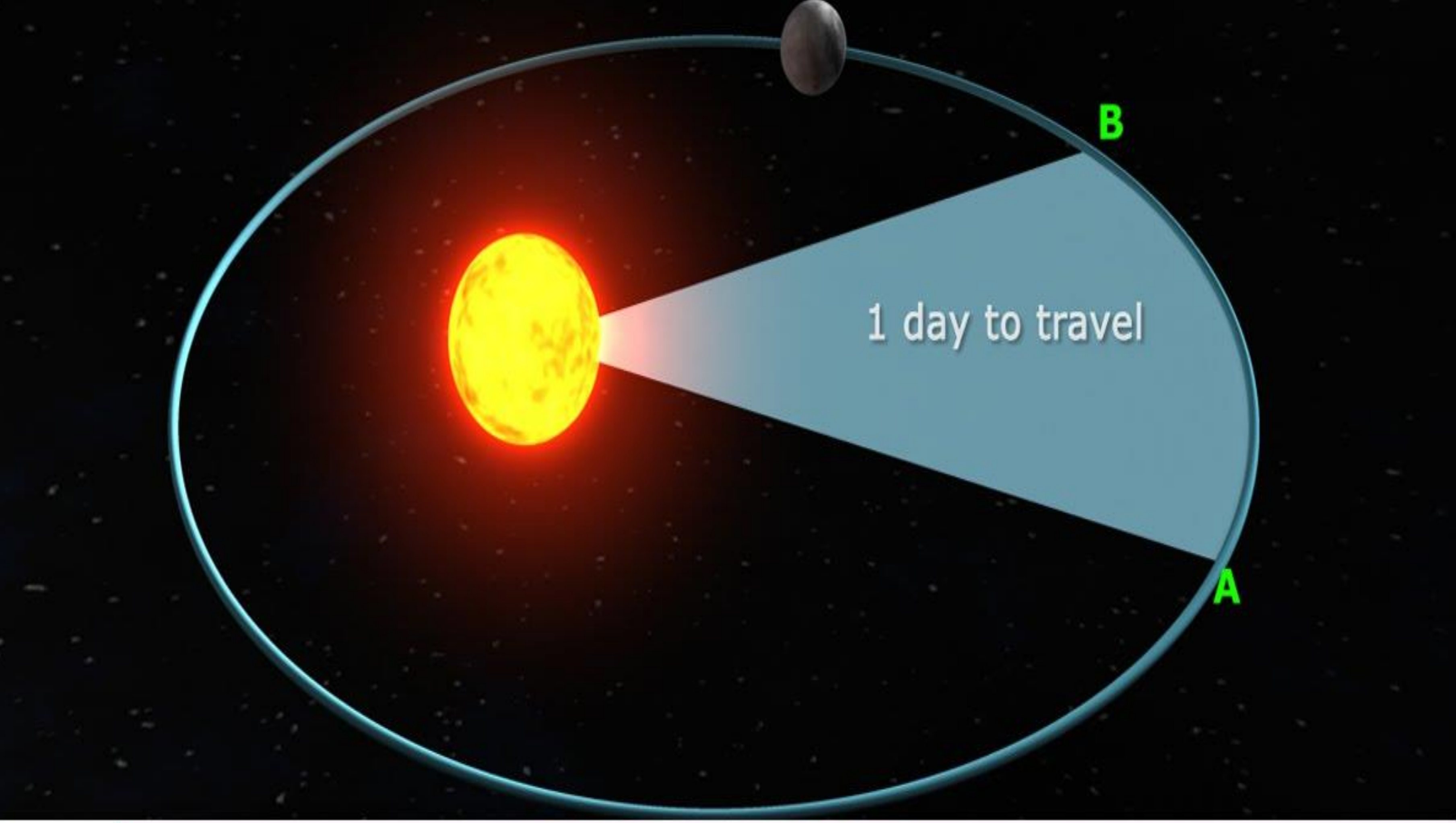
Third Law

(law of harmonics): The square of a planet's orbital time is proportional to its average distance from the star (Sun) cubed.

Second law

A radius vector joining any planet to the Sun sweeps out equal areas in equal lengths of time.





ACTIVE
Happy Learning

KEPLER'S LAW OF PLANETARY MOTION

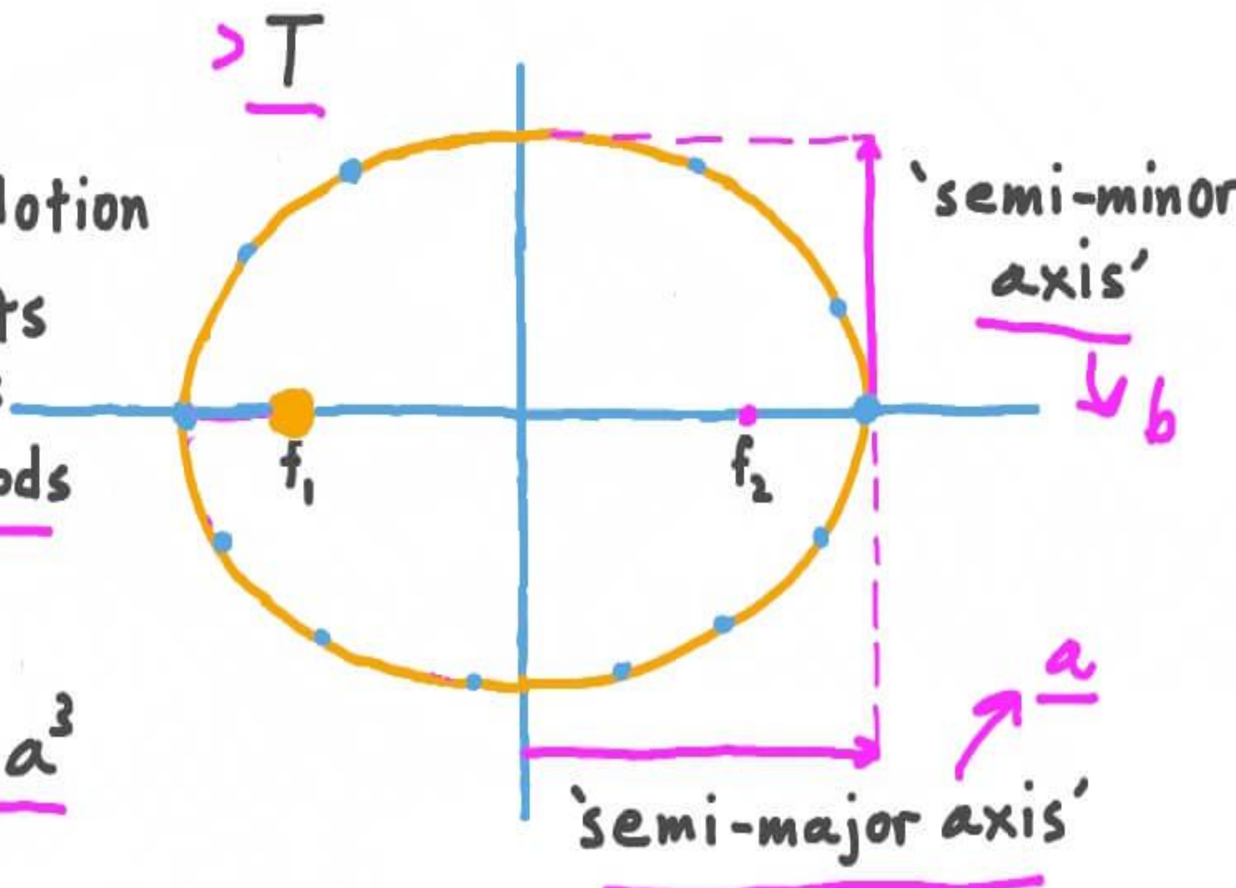
Kepler's Laws of Planetary Motion

- 3 Laws of Planetary Motion

1. Law of Orbits
2. Law of Areas
3. Law of Periods

$$T^2 \propto a^3$$

$$T^2 = \frac{4\pi^2}{G\,M} a^3$$



Johannes Kepler, working with data painstakingly collected by Tycho Brahe without the aid of a telescope, developed three laws which described the motion of the planets across the sky.

1. The Law of Orbits: All planets move in elliptical orbits, with the sun at one focus.
2. The Law of Areas: A line that connects a planet to the sun sweeps out equal areas in equal times.
3. The Law of Periods: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

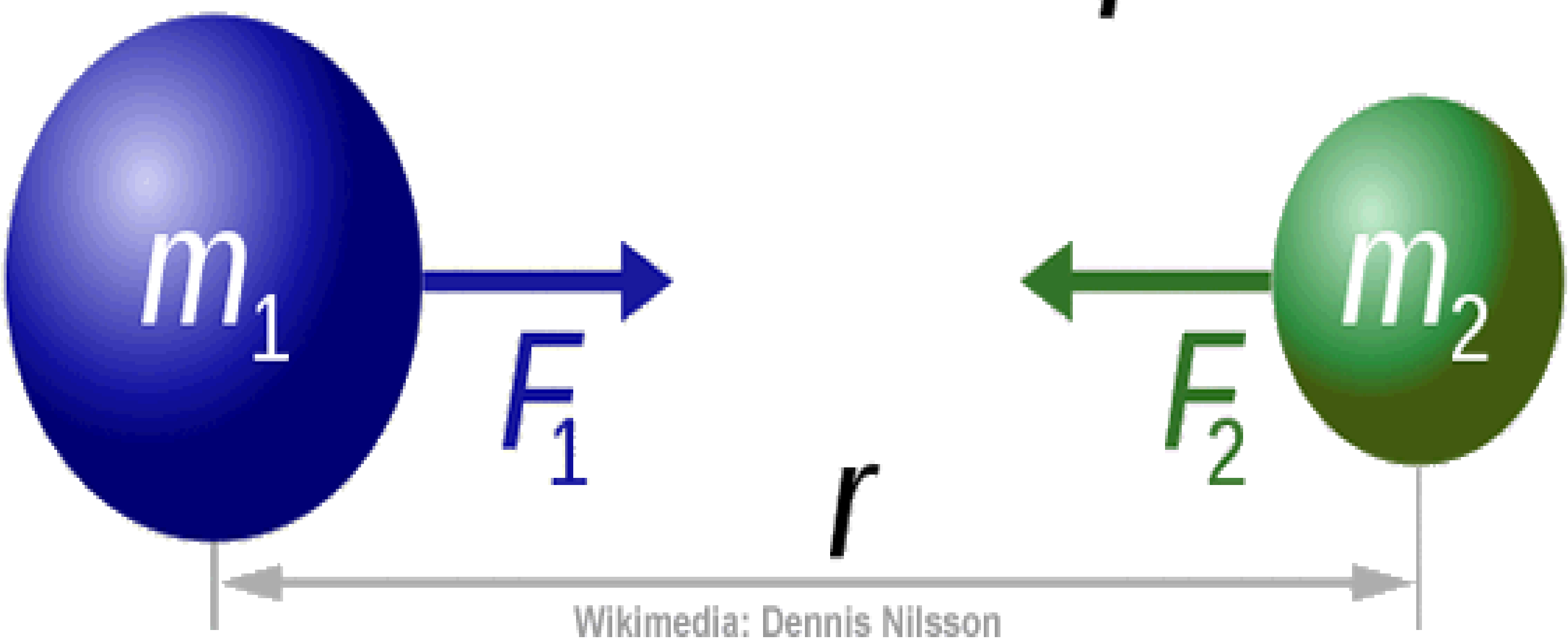
Kepler's laws were derived for orbits around the sun, but they apply to satellite orbits as well.

Newton's law of universal gravitation

Newton's law of universal gravitation is usually stated as that every particle attracts every other particle in the universe with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Newton's Law of Universal Gravitation

$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$



Wikimedia: Dennis Nilsson

Law of Universal Gravitation

Every object in the Universe attracts every other object with a force directed along the line of centers for the two objects that is proportional to the product of their masses and inversely proportional to the square of the separation between the two objects.

$$F_g = G\frac{m_1 m_2}{r^2}$$

F_g is the gravitational force

m_1 & m_2 are the masses of the two objects

r is the separation between the objects

G is the universal gravitational constant

Newton's Law of Gravitation

- Every particle of matter attracts every other particle of matter with a force directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$\mathbf{F} = \frac{-Gm_1 m_2}{r_{12}^{\;2}}\,\hat{\mathbf{r}}$$

$G = 6.67 \times 10^{-11}\ \mathrm{m^3\,s^{-2}\,kg^{-1}}$ is the *gravitational constant*

Gravitational Force

- Newton's universal law of gravitation
  Every object in the universe attracts every other object with a force that is directly proportional to the mass of each body and that is inversely proportional to the square of the distance between them.

$$F_g = G\,\frac{m_1 m_2}{d^2} \qquad \text{Where } G = 6.67 \times 10^{-11}\,\frac{\text{N m}^2}{\text{kg}^2}$$

Universal Gravitational Constant

$$F = G\frac{m_1 m_2}{r^2}; G = 6.67 \times 10^{-11}(SI).$$

The proportionality constant, G is called the universal gravitational constant. Its value in the SI system of units is, $G = 6.67 \times 10^{-11} N.m^2/Kg^2$.

The law of gravitation is universal and very fundamental. It can be used to understand the motions of planets and moons, determine the surface gravity of planets, and the orbital motion of artificial satellites around the Earth.

G vs. g

- Obviously, it's very important to distinguish between **G** and **g**
  - They are obviously very different physical quantities!

G ≡ <u>The Universal Gravitational Constant</u>

It is the same everywhere in the Universe!

$$\mathbf{G = 6.673 \times 10^{-11}\ N{\cdot}m^2/kg^2}$$

It ***<u>ALWAYS</u>*** has this value at every location anywhere

g ≡ The Acceleration Due to Gravity

$\mathbf{g = 9.80\ m/s^2}$ (approximately!) on Earth's surface

g varies with location

$$F \propto M \times m \qquad \text{(i)}$$

$$F \propto {}^{1}\!/\!{}_{d^2} \qquad \text{(ii)}$$

Combine (i) and (ii)

$$F \propto \frac{M \times m}{d^2}$$

$$F = G\,\frac{M \times m}{d^2}$$

G is universal gravitation constant.

For example, consider an object of mass 80 kg standing 10 mm from another object with a mass of 65kg. The attractive gravitational force between them would be:

$$F=Gm1m2/d^2$$

$$= (6.67\times\ 10^{-11})((80)(65)(10)2)$$

$$=3.47\times 10^{-9}\ N$$

As you can see, these forces are very small. Now consider the gravitational force between the Earth and the Moon. The mass of the Earth is $5.98x10^{24}\ kg$ $7.35x10^{22}\ kg$ $3.8x10^{8}mm$

$$F=Gm1m2/d^2$$

$$= \frac{(6.67\times\ [\![10]\!]\text{^}(-11))(5.98x10^{24}\ kg\ \ x\ 7.35x10^{22}\ kg}{(3.8x10^{8}mm)\text{^}2}$$

$$F=2{,}03\times 10^{20}\ N$$

Deduction of Newton’s Law of Gravitation from Kepler’s Law

Newton’s Law of Gravitation is states that in this universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the product of the squares of the distance between them.

Newton’s Law of Gravitation can be easily obtained from Kepler’s Laws of Planetary Motion.

Suppose a planet of mass m is revolving around the sun of mass M in a nearly circular orbit of radius r, with a constant angular velocity ω. Let, T be the time period of revolution of the planet around the sun. $\omega=2\pi/T$ ... (1). The centripetal force acting on the planet for its circular motion is:

$$F_1 = M_1 R_1 \omega_1{}^2 = M_1 R_1 \left[\frac{2\pi}{T_1}\right]^2 \quad (2)$$

$$F_2 = M_2 R_2 \omega_2{}^2 = M_2 R_2 \left[\frac{2\pi}{T_2}\right]^2 \quad (3)$$

so eqn(2)/(3)

$$\frac{F_1}{F_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_1}{R_2}\right)\left(\frac{T_2}{T_1}\right)^2$$

According to Kepler’s Third Law: $T^2 \alpha R^3$ $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$

$$\frac{F_1}{F_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_1}{R_2}\right)\left(\frac{R_2}{R_1}\right)^3 = (m/m)\left(\frac{M_1}{M_2}\right)\left(\frac{R_2}{R_1}\right)^2 = \left(\frac{mM_1}{mM_2}\right)\left(\frac{R_2}{R_1}\right)^2$$

$$F_1 = \frac{mM_1}{R_1{}^2} \quad \text{and} \quad F_2 = \frac{mM_2}{R_2{}^2}$$

This centripetal force is provided by the gravitational attraction exerted by the sun on the planet. According to Newton, the gravitational attraction between the sun and the planet is mutual. If force F is directly proportional to the mass of the planet (m). It should be directly proportional to the mass of the sun (M) inversely proportional to the square of the distance from the sun.

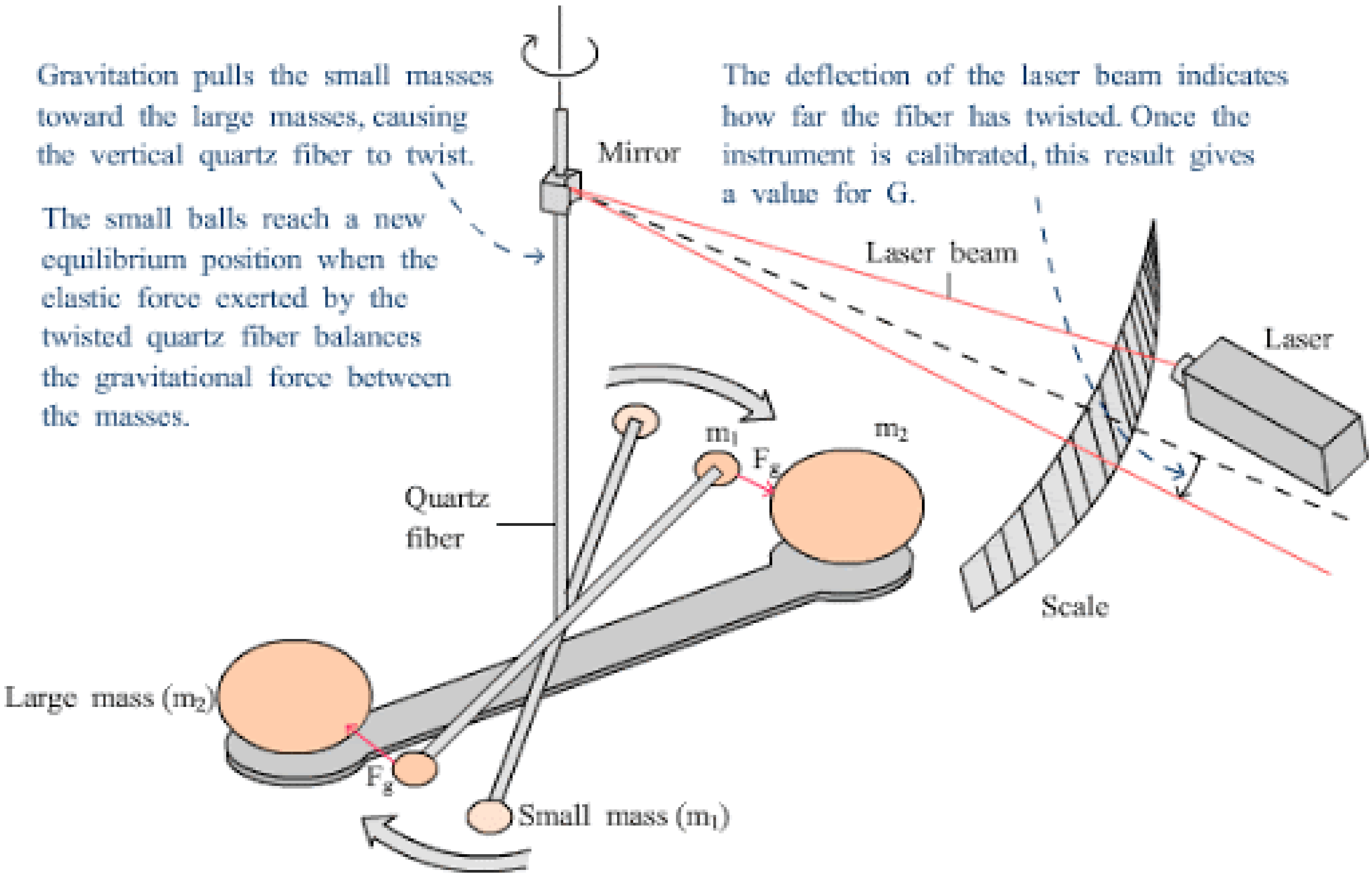
On the basis of Kepler's Laws, Newton concluded the following:

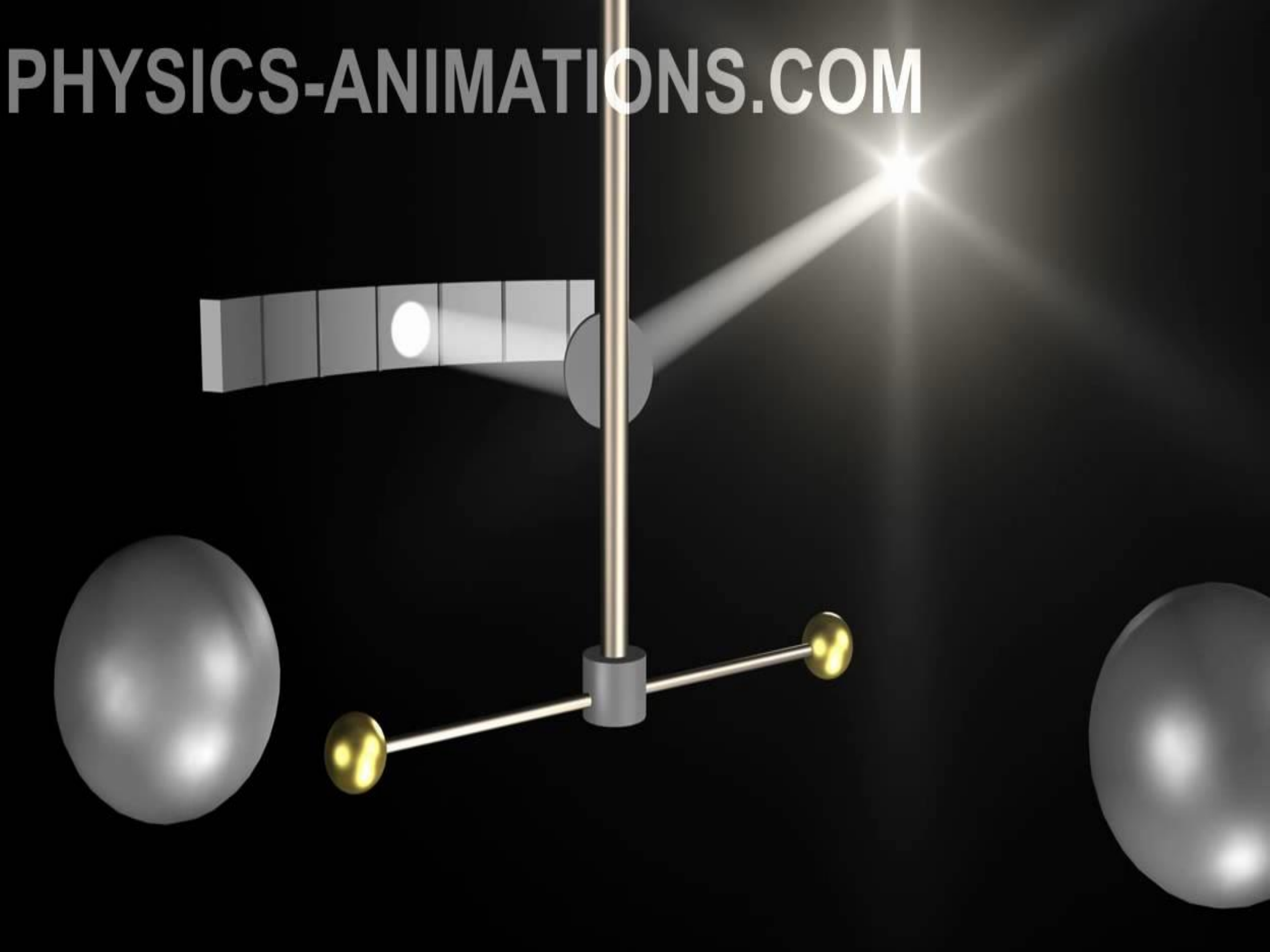
1. A force acting on the planet due to sun is the centripetal force which is directed towards the sun.
2. The force acting on the planet must be inversely proportional to the square of the distance from the sun.
3. The force acting on the planet is directly proportional to the product of the masses of the planet and the sun.

Inertial mass	Gravitational mass
F=ma	F=mg =w
m=F/a	m=F/g

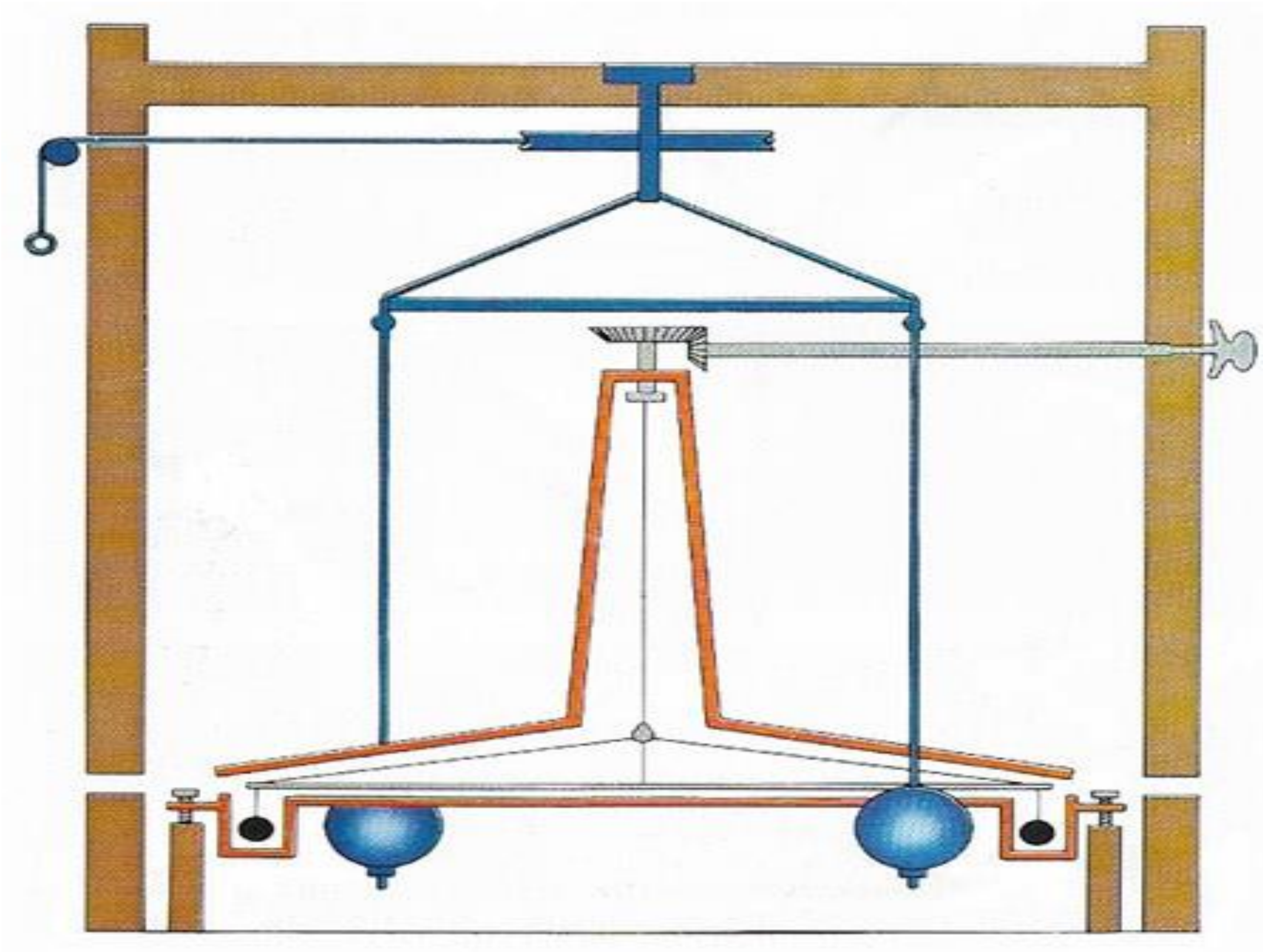
	mass	weight
1	m	w=mg
2	constant	variable
3	scalar	vector
4	how much matter in how volume	Force

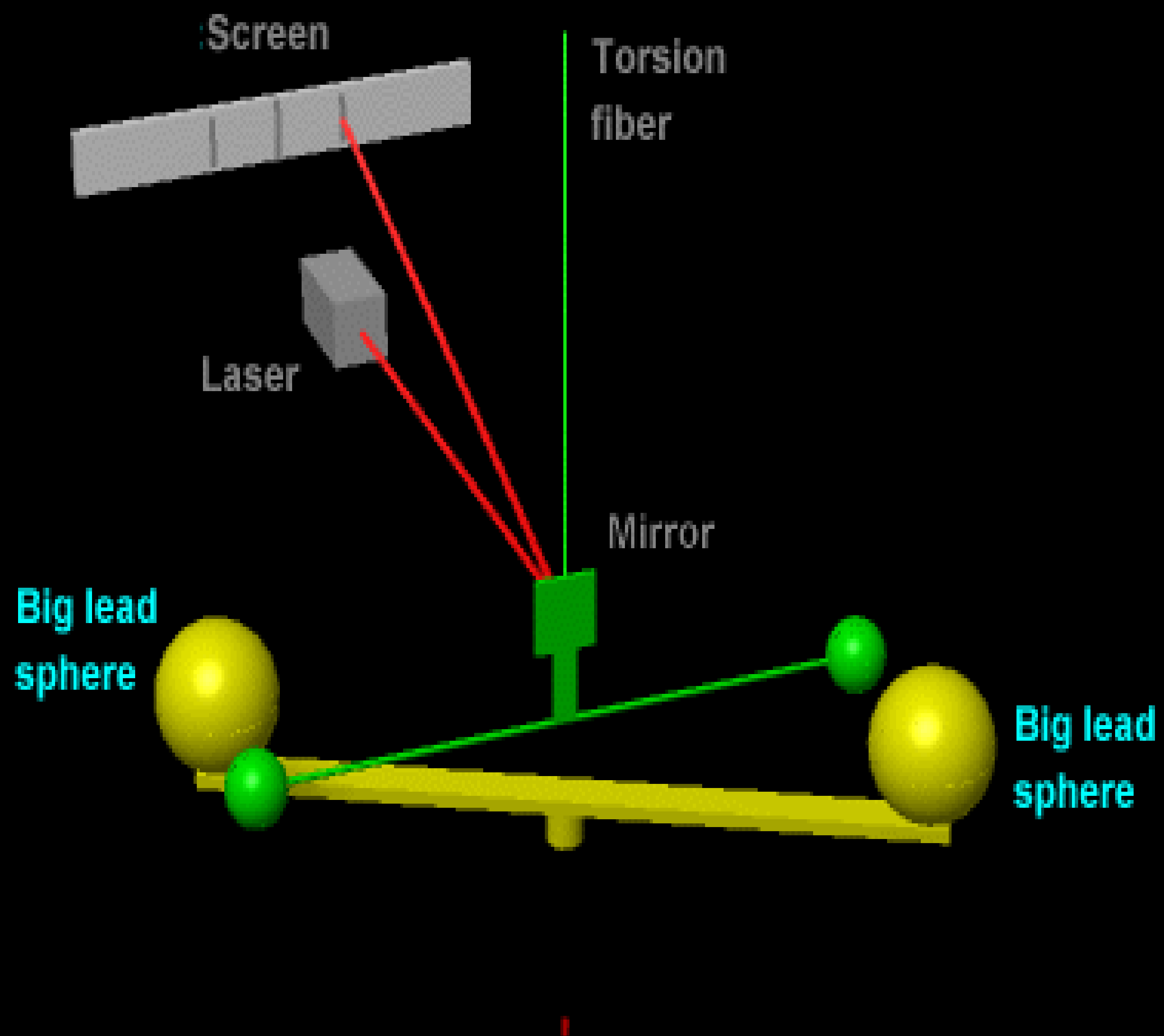
Boy's method of determining gravitational constant G

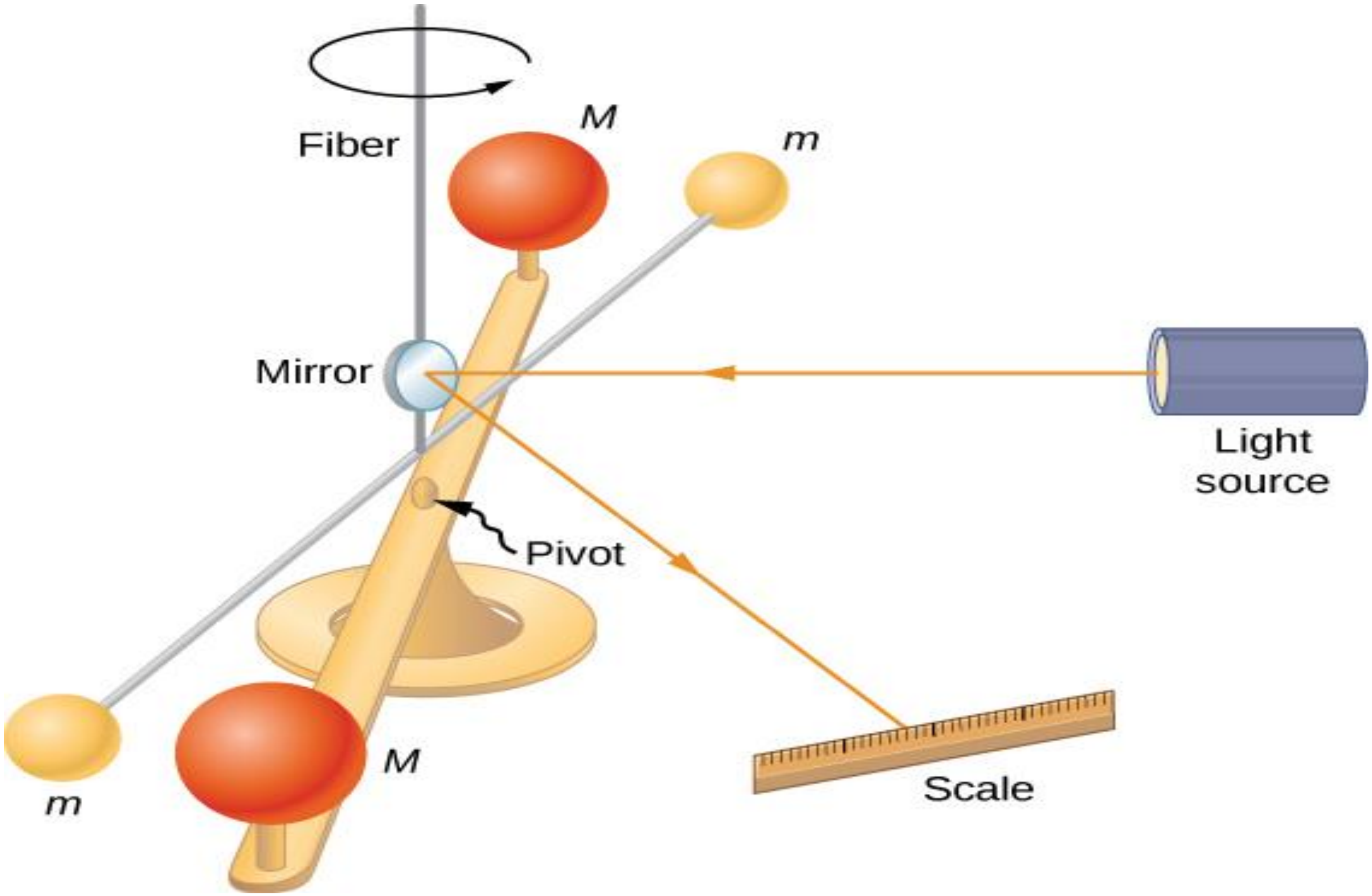



PHYSICS-ANIMATIONS.COM

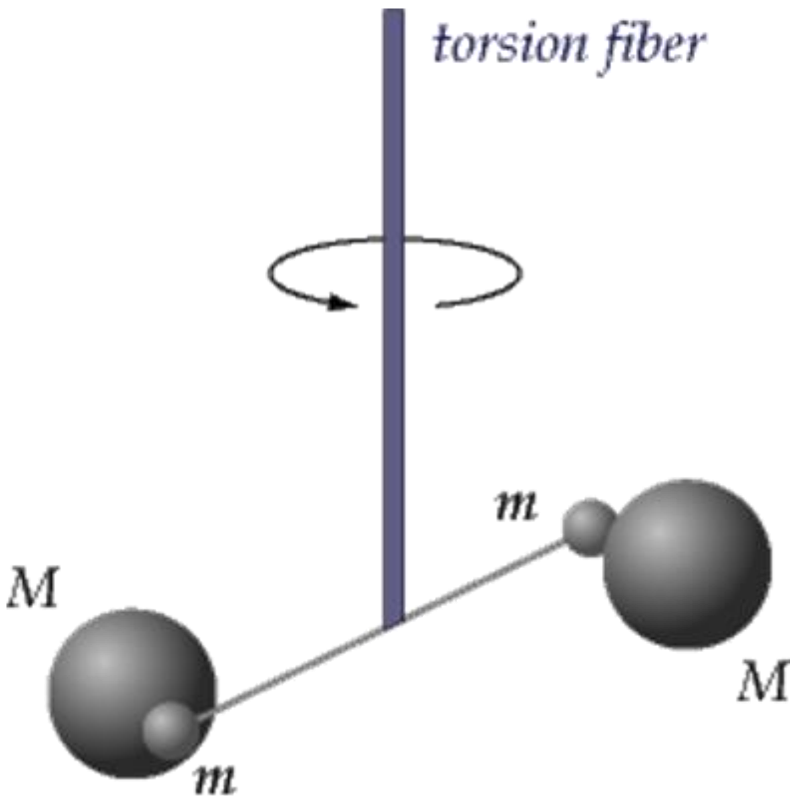
Boy’s method of determining gravitational constant G

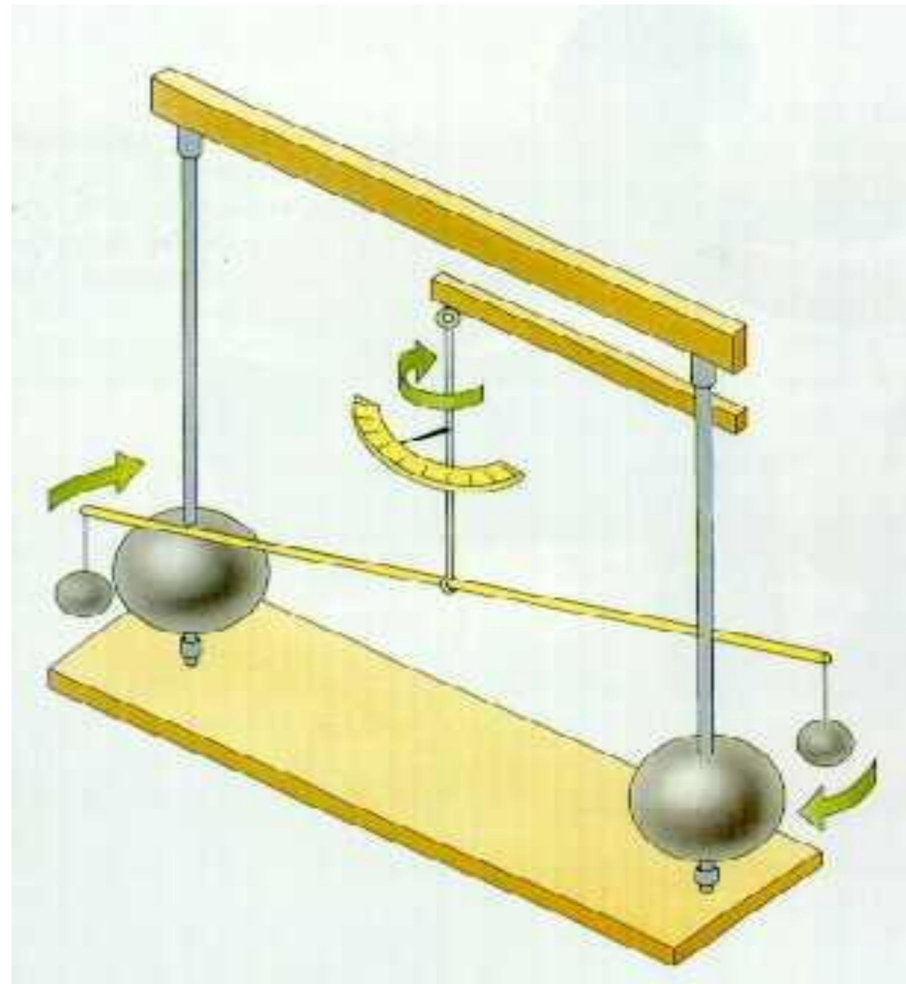



Screen
Torsion
fiber
Laser
Mirror
Big lead
sphere
Big lead
sphere



Fiber
M
m
Mirror
Light
source
Pivot
m
M
Scale

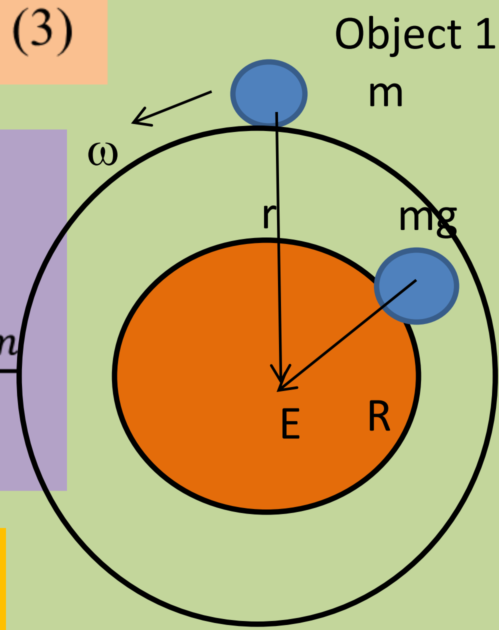

torsion fiber
m
M
M
m



Centripetal or centrifugal force $= \frac{mv^2}{r} = \frac{m(r^2\omega^2)}{r} = mr\omega^2$ (1)

Weight or force due to acceleration due to gravity = mg (2)

Gravitational force $= \frac{GMm}{r^2}$ (3)



So comparing we get,

Centripetal or centrifugal force = weight so $\frac{mv^2}{r} = mr\omega^2 = mg$

Centripetal or centrifugal force = Gravitational force $mr\omega^2 = \frac{GMm}{r^2}$

$$\frac{mv^2}{r} = mr\omega^2 = mg$$

$$mr\omega^2 = \frac{GMm}{r^2}$$

$$\frac{v^2}{r} = g \text{ or } r\omega^2 = g$$

$$\frac{v^2}{r} = \frac{GM}{r^2} \text{ or } r\omega^2 = \frac{GM}{r^2}$$

$$mg = \frac{GMm}{r^2} \text{ so } g = \frac{GM}{r^2}$$

$$r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = g$$

$$r\,\frac{4\pi^2}{T^2} = g$$

$$r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^2}$$

From equation

$$g = \frac{GM}{R^2}$$

From equation (2.3)

$$g_h = \frac{GM}{(R + h)^2}$$

$$\therefore \quad \frac{g_h}{g} = \frac{R^2}{(R + h)^2}$$

$$\therefore \quad g_h = \frac{gR^2}{(R + h)^2} \qquad \text{...(2.20)}$$

From this equation it is obvious that acceleration due to gravity goes on decreasing as altitude of body from the earth's surface increases.

$$\frac{g_h}{g} = \frac{R^2}{(R + h)^2}$$

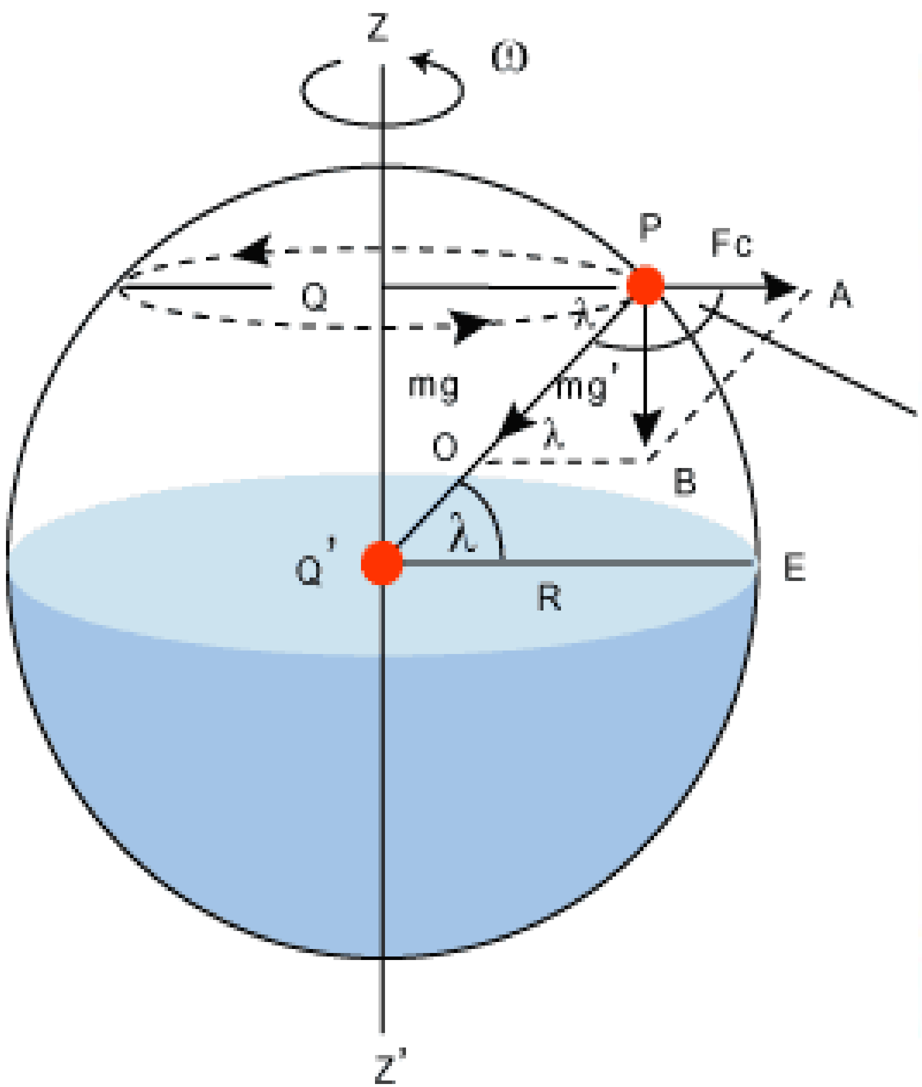
$$= \frac{R^2}{R^2\left[1 + \frac{h}{R}\right]^2} = \left[1 + \frac{h}{R}\right]^{-2}$$

$$\therefore \quad \frac{g_h}{g} = \left[1 - \frac{2h}{R}\right]$$

higher powers of $\frac{h}{R}$ are neglected. (here we ssume that $\frac{h}{R} << 1$) we get

$$\therefore \quad g_h = g\left[1 - \frac{2h}{R}\right] \qquad \text{...(2.21)}$$

Variation of g with latitude or rotation of Earth



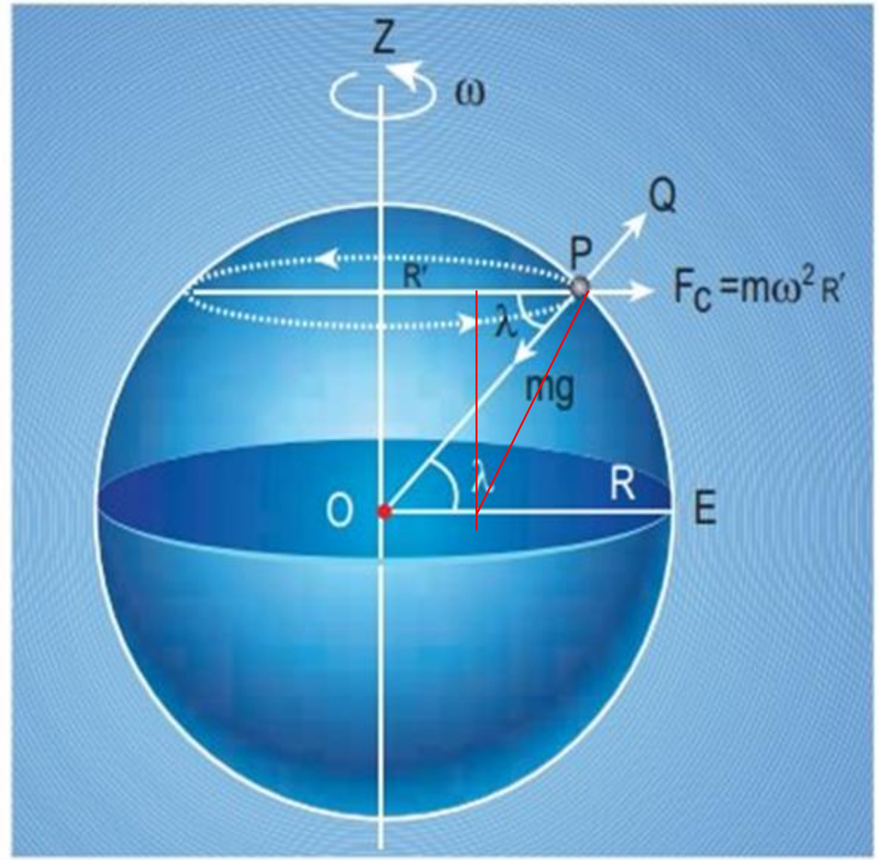


Figure 6.18 Variation of g with latitude

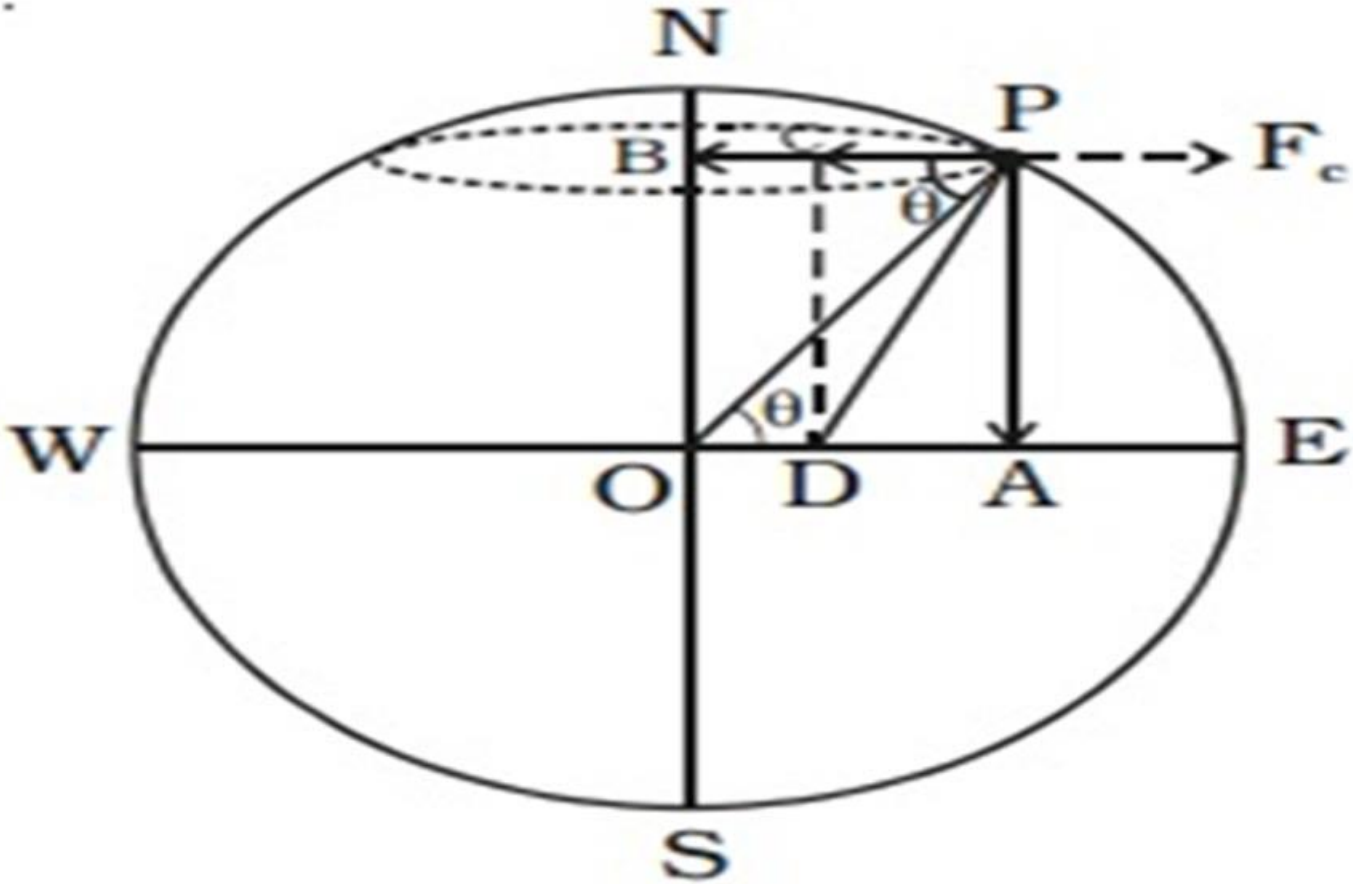
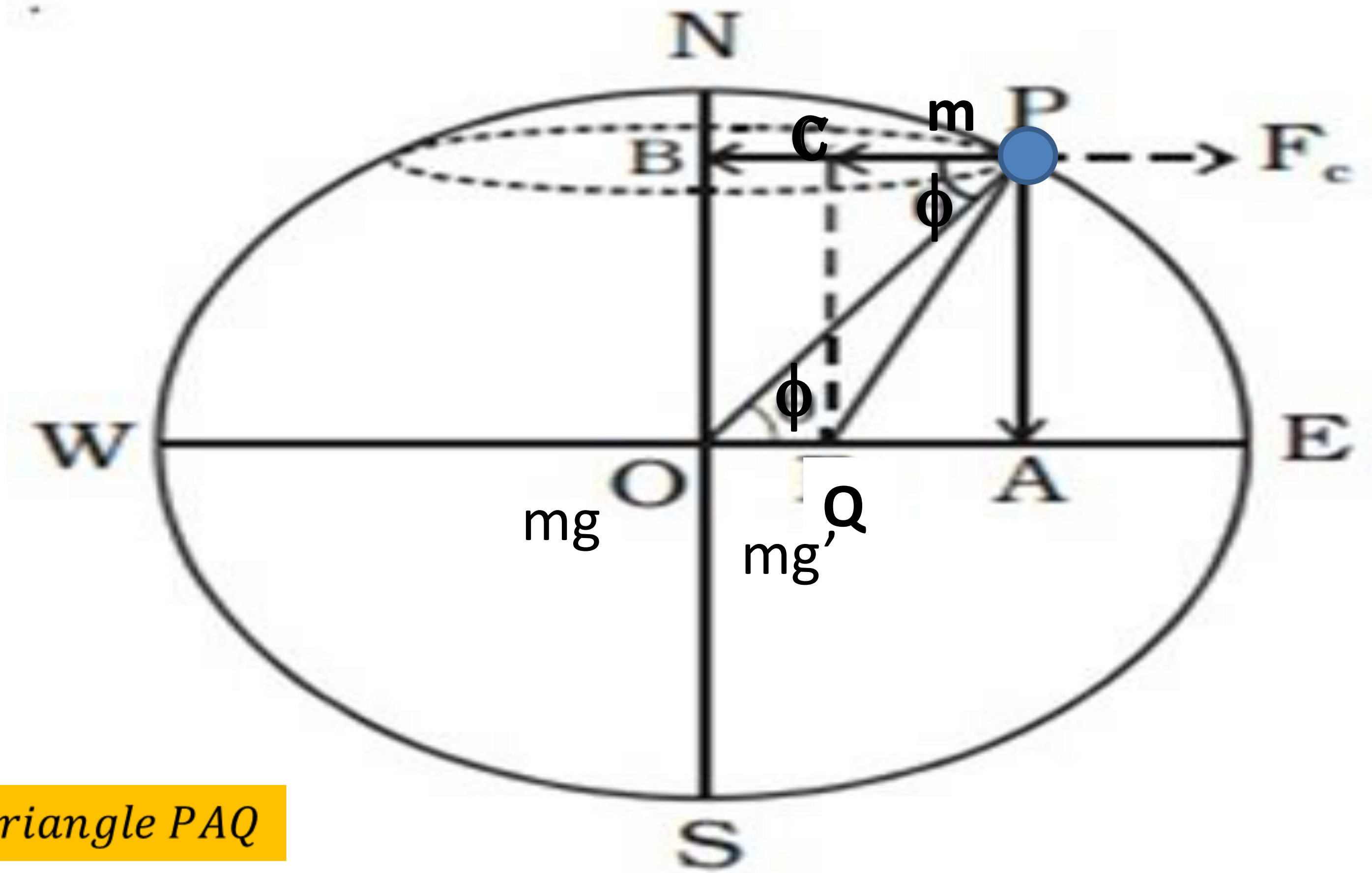


Fig. Rotation of the Earth



from triangle PAQ

$$PQ^2 = PA^2 + QA^2$$

$$= CQ^2 + PC^2$$

$$(mg')^2 = (mgSin\emptyset)^2 + (mgCos\emptyset - \mathrm{mRCos}\emptyset\omega^2)^2 \quad \text{Eqn A}$$

$$mg' = \sqrt{(mgSin\emptyset)^2 + (mgCos\emptyset - \mathrm{mRCos}\emptyset\omega^2)^2}$$

Variation of g with latitude (Rotation of the Earth)

Let us consider the Earth as a homogeneous sphere of mass M and radius R. The Earth rotates about an axis passing through its north and south poles. The Earth rotates from west to east in 24

Consider a body of mass m on the surface of the Earth at P at a latitude θ. Let ω be the angular velocity. The force (weight) F = mg acts along PO. It could be resolved into two rectangular components (i) mg cos θ along PB and (ii) mg sin θ along PA (Fig.).

From the ΔOPB, it is found that BP = R cos ϕ. The particle describes a circle with B as centre and radius BP = R cos θ.

The body at P experiences a centrifugal force (outward force) F_C due to the rotation of the Earth

(i.e) $F_C = mR\omega^2 \cos\theta$, The net force along PC = $mg \cos\theta - mR\omega^2 \cos\theta$

The body is acted upon by two forces along PA and PC. The resultant of these two forces is from eqn A (from previous slide)$(mg')^2$=$(mgSin\emptyset)^2$+$(mgCos\emptyset - \mathrm{mRCos}\emptyset\omega^2)^2$

$$F = mg' = \sqrt{(mgSin\emptyset)^2 + (mgCos\emptyset - \mathrm{mRCos}\emptyset\omega^2)^2}$$

$$mg' = mg\left((Sin\emptyset)^2 + \left(Cos\emptyset - \frac{\mathrm{RCos}\emptyset\omega^2}{g}\right)^2\right)^{1/2}$$

$$mg' = mg\left((Sin\emptyset)^2 + (Cos\emptyset)^2 + \frac{R^2Cos\emptyset^2\omega^4}{g^2} - \frac{2\mathrm{RCos}\emptyset^2\omega^2}{g}\right)^{1/2}$$

$since\ \frac{R^2Cos\emptyset^2\omega^4}{g^2}$ is negligibly small and $(Sin\emptyset)^2 + Cos\emptyset^2$ =1

$$g' = g\left(1 - \frac{2\mathrm{RCos}\emptyset^2\omega^2}{g}\right)^{1/2}$$

$$g' = g\left(1 - \frac{2\mathrm{RCos}\phi^2\omega^2}{g}\right)^{1/2}$$

Expanding and neglecting higher power terms

$$g' = g\left(1 - \frac{\mathrm{RCos}\phi\omega^2}{g}\right)$$

$g' = g\ (1\text{-}(R\omega^2\cos^2\theta\ /g))$

Case (i) At the poles, $\theta = 90^o$; $\cos\theta = 0$

$g' = g$

Case (ii) At the equator, $\theta = 0$; $\cos\theta = 1$

$g' = g(1\text{-}\ R\omega^2/g)$

So, the value of acceleration due to gravity is maximum at the poles.

Variation of g with latitude (Rotation of the Earth)

Let us consider the Earth as a homogeneous sphere of mass M and radius R. The Earth rotates about an

ast in 24 hours.

Consider a body of mass m on the surface of the Earth at P at a latitude θ. Let ω be the angular velocity. The force (weight) F = mg acts along PO. It could be resolved into two rectangular components (i) mg cos θ along PB and (ii) mg sin θ along PA (Fig.).

From the ΔOPB, it is found that BP = R cos ϕ. The particle describes a circle with B as centre and radius BP = R cos θ.

The body at P experiences a centrifugal force (outward force) F_C due to the rotation of the Earth

(i.e) $F_C = mR\omega^2 \cos\theta$, The net force along PC = $mg\cos\theta - mR\omega^2\cos\theta$

The body is acted upon by two forces along PA and PC. The resultant of these two forces is from eqn A (from previous slide)

$(mg')^2 = (mgSin\emptyset)^2 + (mgCos\emptyset - mRCos\emptyset\omega^2)^2$

F= $\sqrt{}$(mg sinθ) ^2+(mg cosθ−mRω^2 cosθ)^2

The force, F = mg g $\sqrt{}$[2Rω^2cos$^2\theta$ /g] 1)

f g′ is the acceleration of the body at P due to this force F, we have,

F = mg′ (2)

by equating (2) and (1)

g′ = g (1-(Rω^2cos$^2\theta$ /g))

Case (i) At the poles, $\theta = 90^o$; cos θ = 0

g′ = g

Case (ii) At the equator, θ = 0 ; cos θ = 1

g′ = g(1- Rω^2/g)

So, the value of acceleration due to gravity is maximum at the poles.

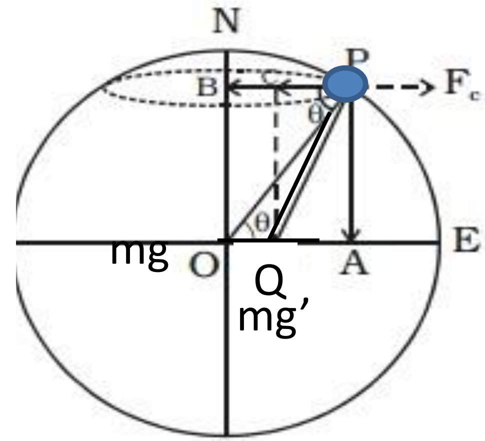


Fig. Rotation of the Earth

Gravitational field and potential

GRAVITATIONAL FIELD

- A gravitational field is a region where a mass experiences a force due to gravitational attraction.
- Gravitational field strength, g, is defined as force per unit mass.

The region of space surrounding a body in which another body experiences a force of gravitational attraction.
A field is something that has a magnitude and a direction at every point in space. There is an **acceleration due to gravity** of about 9.8 m/s2 down at every point in the room. Or **the magnitude of the Earth's gravitational field** is 9.8 m/s2 down at all points in this room.
Gravitational field: g = F/m where F is the force of gravity.

Gravitational field strength (g)

This is equal to the force that acts on a very small unit test mass

Definition: $g = \dfrac{\text{force}}{\text{mass}}$ $\qquad g = \dfrac{F}{m}$

unit of g: N kg $^{-1}$

VECTOR: Direction the same as the force

Earth, sea level average: g = 9.81 N kg^{-1}

Moon, surface average: g = 1.6 N kg^{-1}

Gravitational field

An area or region where a mass feels a gravitational force is called a gravitational field.

The gravitational field strength at any point in space is <u>defined</u> as the force per unit mass (on a small test mass) at that point.

g = F/m (in N.kg^{-1})

The Gravitational Field, 2

When a particle *(test particle)* of mass m is placed at a point where the ***gravitational field*** is $\mathbf{g}$, the particle experiences a force: $\mathbf{F}_g = m\mathbf{g}$

The ***field*** exerts a force **on** the *particle*

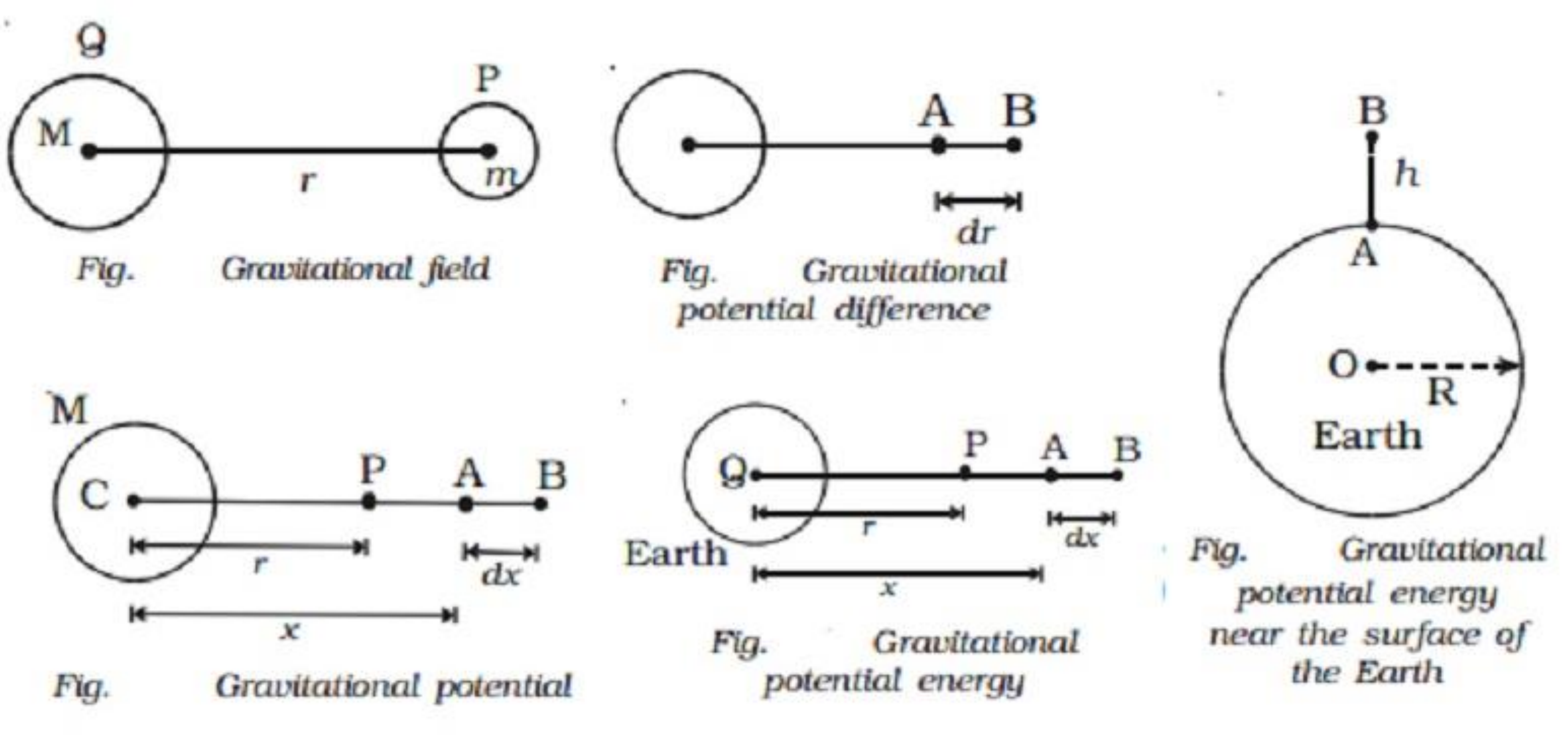
The ***gravitational field*** $\mathbf{g}$ is defined as

$$\mathbf{g} \equiv \frac{\mathbf{F}_g}{m}$$

Where,

F=GmM/r^2

The ***gravitational field*** is the gravitational force experienced by a *test particle* placed at that point divided by the mass of the *test particle*



- In physics, a gravitational field is a model used to explain the influence that a massive object extends into the space around itself, producing a force on another massive object. Thus, a gravitational field is used to explain gravitational phenomena, and is measured in Newtons per kilogram.

Gravitational field

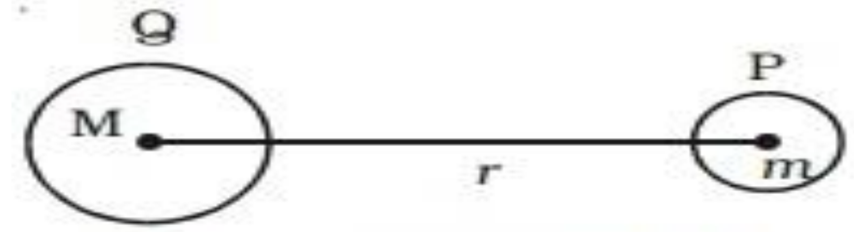


Fig. Gravitational field

Two masses separated by a distance exert gravitational forces on one another. They interact even though they are not in contact. This interaction can also be explained with the field concept. A particle or a body placed at a point modifies a space around it which is called gravitational field. When another particle is brought in this field, it experiences gravitational force of attraction.

The gravitational field is defined as the space around a mass in which it can exert gravitational force on other mas.

Gravitational field intensity

Gravitational field intensity or strength at a point is defined as the force experienced by a unit mass placed at that point. It is denoted by E. It is a vector quantity. Its unit is N kg-1.

Consider a body of mass M placed at a point Q and another body of mass m placed at P at a distance r from Q.

The mass M develops a field E at P and this field exerts a force $F = mE$.

The gravitational force of attraction between the masses m and M is $F = GMm/r2$

The gravitational field intensity at P is $E = F/m$

$E = GM/r^2$

Gravitational field intensity is the measure of gravitational field.

Gravitational potential difference

Gravitational potential difference between two points is defined as the amount of work done in moving unit mass from one point to another point against the gravitational force of attraction.

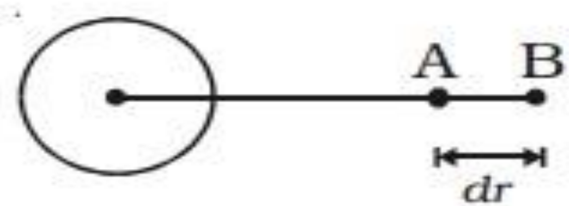


Fig. Gravitational potential difference

Consider two points A and B separated by a distance dr in the gravitational field.

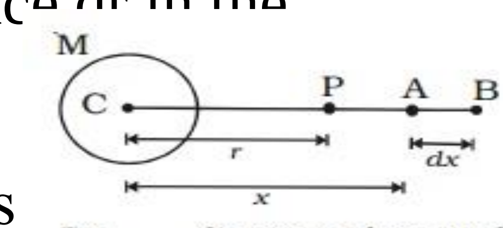


Fig. Gravitational potential

The work done in moving unit mass from A to B is

Gravitational potential difference $dv = -E\,dr$

Here negative sign indicates that work is done against the gravitational field.

Gravitational potential

Gravitational potential at a point is defined as the amount of work done in moving unit mass from the point to infinity against the gravitational field. It is a scalar quantity. Its unit is N m kg-1.

Expression for gravitational potential at a point

- Consider a body of mass M at the point C. Let P be a point at a distance r from C.

To calculate the gravitational potential at P consider two points A and B. The point A, where the unit mass is placed is at a distance x from C.

The gravitational field at A is $E = GM/x^2$

The work done in moving the unit mass from A to B through a small distance dx is $dw = dv = -E.dx$

Negative sign indicates that work is done against the gravitational field. $dv = -(GM/x^2)\, dx$

The work done in moving the unit mass from the point P to infinity is $\int dv = - \int (GM/x2)\, dx$

$v = - GM/r$

The gravitational potential is negative, since the work is done against the field. (i.e) the gravitational force is always attractive.

Gravitational potential energy

Consider a body of mass m placed at P at a distance r from the centre of the Earth. Let the mass of the Earth be M.

When the mass m is at A at a distance x from Q, the gravitational force of attraction on it due to mass M is given by F = GMm / x2

The work done in moving the mass m through a small distance dx from A to B along the line joining the two centres of masses m and M is dw = -F.dx.

Negative sign indicates that work is done against the gravitational field.

dw = - GMm / x2 dx

The gravitational potential energy of a mass m at a distance r from another mass M is defined as the amount of work done in moving the mass m from a distance r to infinity.

The total work done in moving the mass m from a distance r to infinity is

*U = GMm/r

Gravitational potential energy is zero at infinity and decreases as the distance decreases. This is due to the fact that the gravitational force exerted on the body by the Earth is attractive. Hence the gravitational potential energy U is negative.

Gravitational potential energy near the surface of the Earth
Let the mass of the Earth be M and its radius be R. Consider a point A on the surface of the Earth and another point B at a height h above the surface of the Earth.

The work done in moving the mass m from A to B is $U = UB - UA$
$U = GMmh / R(R+h)$

If the body is near the surface of the Earth, h is very small when compared with R. Hence (R+h) could be taken as R.

$U = GMmh / R2$ $\qquad$ $U = mgh$

Law of Gravity

- A description of the gravitational force exerted by one body on another. It is proportional to the product of their masses (M, m) & the inverse square of their separation (d):

$$F_G = G\frac{M \times m}{d^2}$$

$G = 6.67 \times 10^{-11}\ m^3\text{-}kg^{-1}\text{-}s^{-2}$

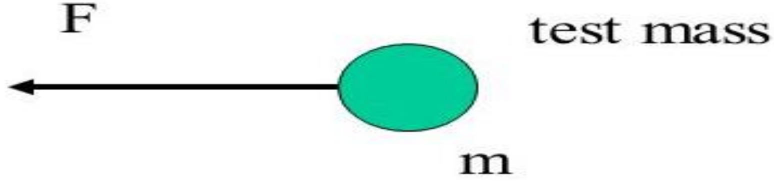
Field strength, g

- The gravitational field strength, g, is the gravitational force per unit mass on a test mass.

$$\vec{g} = \frac{\vec{F}}{m}$$

F is the gravitational force
m is the mass of the test mass

g is a vector, in the same direction of F.
SI unit of g is Nkg^{-1}.



Gravitational Field Strength

- Is defined as the force per unit mass acting on a test mass placed at that point.

$$\vec{E} = \frac{\vec{F}}{m}$$

Its SI unit is NKg^{-1}

Gravitational field strength

Gravitational field strength, g, is defined as the gravitational force per unit mass, so:

$$\boxed{g = F_{grav} / m}$$

By considering the gravitational force on a small mass, m, from a large mass, M, can you derive the following formula for the gravitational field strength in a radial field? $g = GM / r^2$

1. Start with the equation above: $F_{grav} = mg$

2. Use the formula for the force between two point masses:

$$F_{grav} = GmM / r^2$$

3. Rearrange: $mg = GmM / r^2$

4. Cancel m:

$$\boxed{g = GM / r^2}$$

GRAVITATIONAL FIELDS

force between two masses	$F = \dfrac{G m_1 m_2}{r^2}$
gravitational field strength	$g = \dfrac{F}{m}$
magnitude of gravitational field strength in a radial field	$g = \dfrac{GM}{r^2}$
gravitational potential	$\Delta W = m\Delta V$
	$V = -\dfrac{GM}{r}$ $\quad g = -\dfrac{\Delta V}{\Delta r}$

Gravitational Potential Energy

Gravitational potential energy is the energy stored in an object due to its position above the Earth's surface.

$$E_p = mgh$$

m = mass (kg)
g = gravitational field strength (N/kg)
h = height (m)
E_p = gravitational potential energy (J)

Gravitational Potential Energy

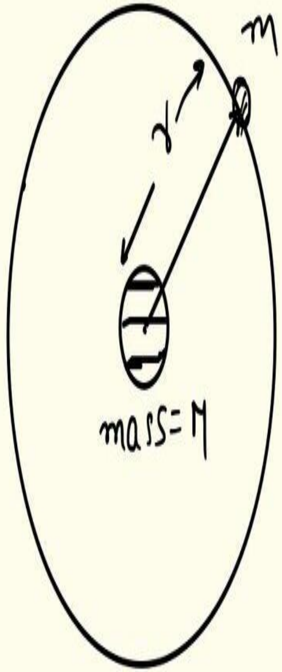
Gravitational potential energy:

energy that is stored in an object due to its height

- An object has the potential to fall
- SI Unit: Joule (J)
- Equation: $PE_G = mgh$
  - PE_G = Gravitational PE (J)
  - M = mass (kg)
  - G = acceleration due to gravity ($10 m/s^2$)
  - H = height (m)

Orbital velocity -

$$V_0 = \sqrt{\frac{GM}{r}}$$



Escape - velocity

$$V_e = \sqrt{\frac{2GM}{r}}$$

$$\therefore \frac{V_e}{V_0} = \sqrt{\frac{2GM}{r}} \times \sqrt{\frac{r}{GM}}$$

$$= \sqrt{2}$$

$$\boxed{V_e = \sqrt{2}\, V_0}$$

$$v_{escape} = 11.2\ km/s$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

* Gravitational potential (V) -:

" The amount of work done in bringing unit mass from infinity to a point in the gravitational field, is defined as gravitational potential at that point.

Let a unit mass be placed at point A, thus gravitational force at A due to M,

$$F = \frac{GM}{r^2} \qquad [m = 1.$$

Let the unit mass is taken from A to B by very small distance dr. then work done against Gravitational force is,

$$dW = F dr = \frac{GM}{r^2} dr$$

$\therefore$ total work done in taken unit mass from A to ∞ against gravitational force is given by

$$W_{r\to\infty} = \int_r^\infty \frac{GM}{r^2} dr = GM \int_r^\infty \frac{1}{r^2} dr$$

$$W_{r\to\infty} = GM \left[-\frac{1}{r}\right]_r^\infty = GM \left[-\frac{1}{\infty} - \left(-\frac{1}{r}\right)\right]$$

$$\Rightarrow \underset{r\to\infty}{W} = \frac{GM}{r} \longrightarrow (1)$$

$\therefore$ Amount of work done in bringing the unit mass from ∞ to r is,

$$\underset{\infty\to r}{W} = -\underset{r\to\infty}{W} = -\frac{GM}{r}$$

i.e $$\boxed{V = -\frac{GM}{r}}$$

Gravitational potential

- Let's assume:
  - A particle of unit mass moving freely
  - A body of mass M
- The particle is attracted by M and moves toward it by a small quantity dr.
- This displacement is the result of **work** W exerted by the gravitational field generated by M:

$$W = Fdr = m \quad a \quad dr = a \quad dr$$

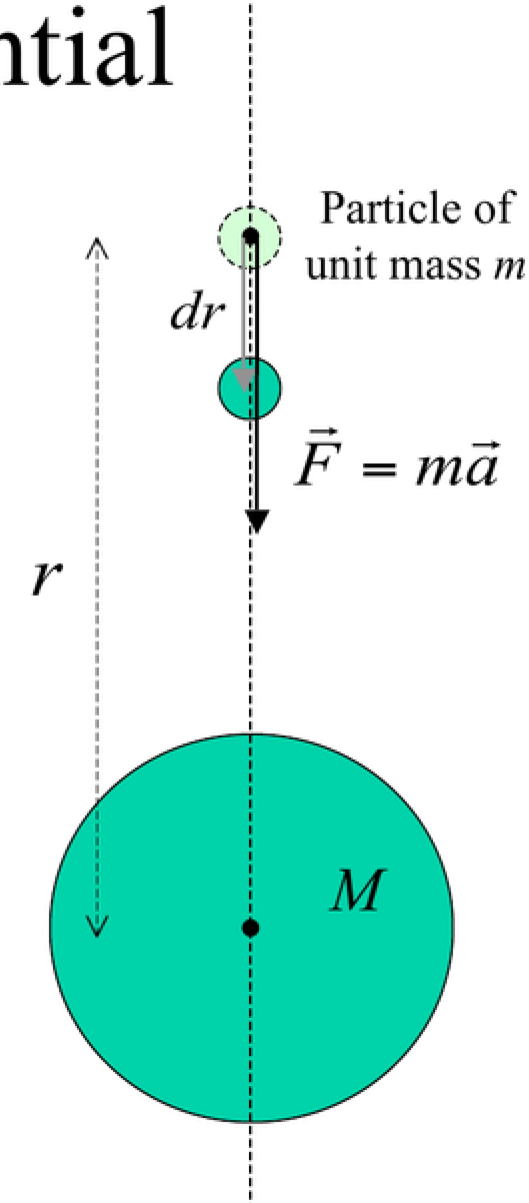
$$\Rightarrow W = G\frac{M}{r^2}dr$$

- The **potential** U of mass M is the amount of work necessary to bring the particle from infinity to a given distance r:

$$U = \int_{\infty}^{r} G\frac{M}{r^2}dr = GM\int_{\infty}^{r}\frac{1}{r^2}dr = GM\left[\frac{1}{\infty} - \frac{1}{r}\right]$$

$$\Rightarrow U = -\frac{GM}{r}$$

- **At distance r, the gravitational potential generated by M is U**



From the definition of potential energy $$U = -\int_{ref}^{r} \vec{F} \cdot \vec{dr}$$

and the law of gravitation $$F = -\frac{GMm}{r^2}\vec{1}_r$$

with the choice of the zero of potential energy at infinite distance where the force approaches zero, the gravitational potential energy is the work done to bring an object from infinity to radius r:

$$U(r) = -\int_{\infty}^{r} -\frac{GMm}{r'^2}\,dr' = -\frac{GMm}{r}$$

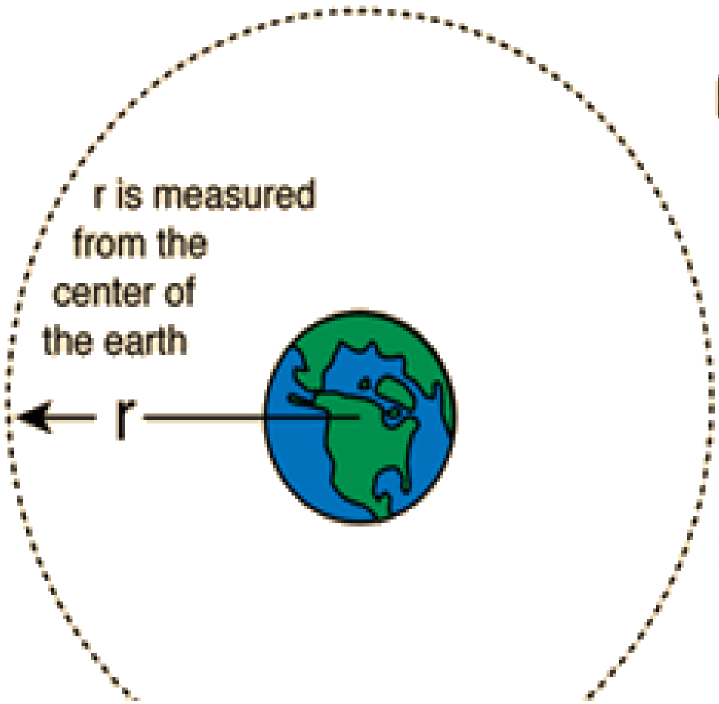
Form of integral

$\vec{1}_r$ represents a unit vector in the outward radial direction

The potential integral is of the polynomial form $\int \frac{1}{r^2}\,dr = -\frac{1}{r}$ so that

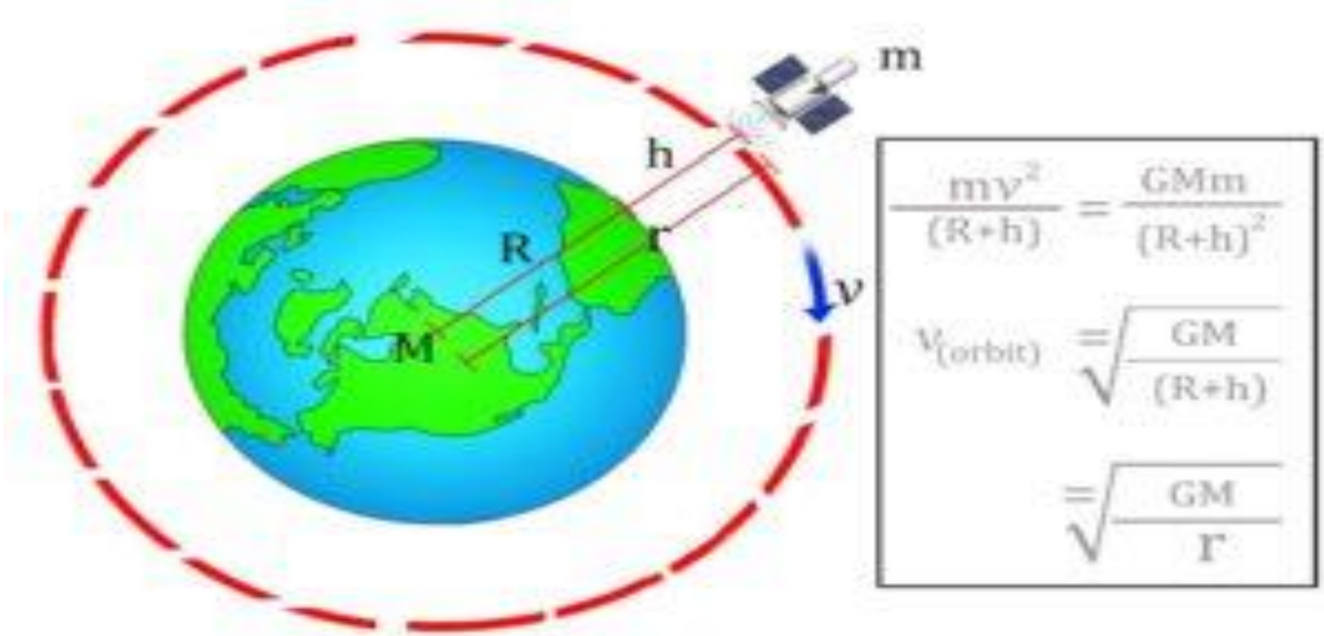
$$U(r) = -\int_{\infty}^{r} -\frac{GMm}{r'^2}\,dr'$$

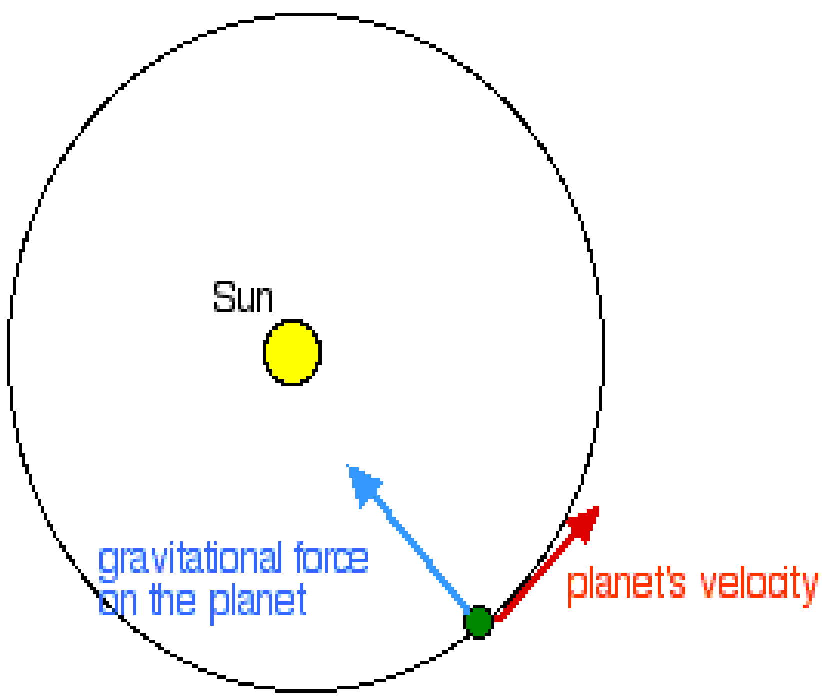
has three minus signs which yield the negative expression $U(r) = -\frac{GMm}{r}$. The negative potential energy indicates a bound state.



$$F_{gravity} = -\frac{GMm}{r^2}$$

An object at radius r out from the earth is bound to the earth by energy U, and would require the supply of extra energy equal to U to escape the earth's gravity.





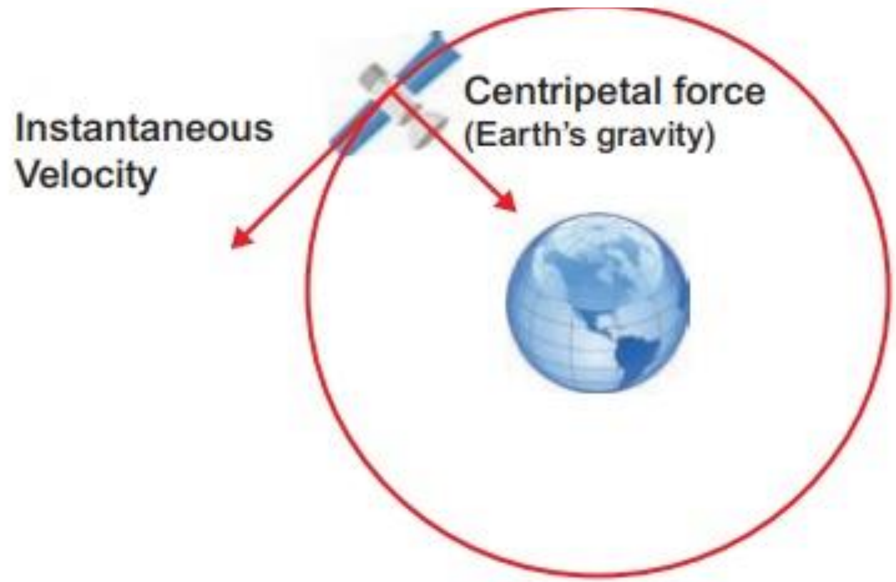
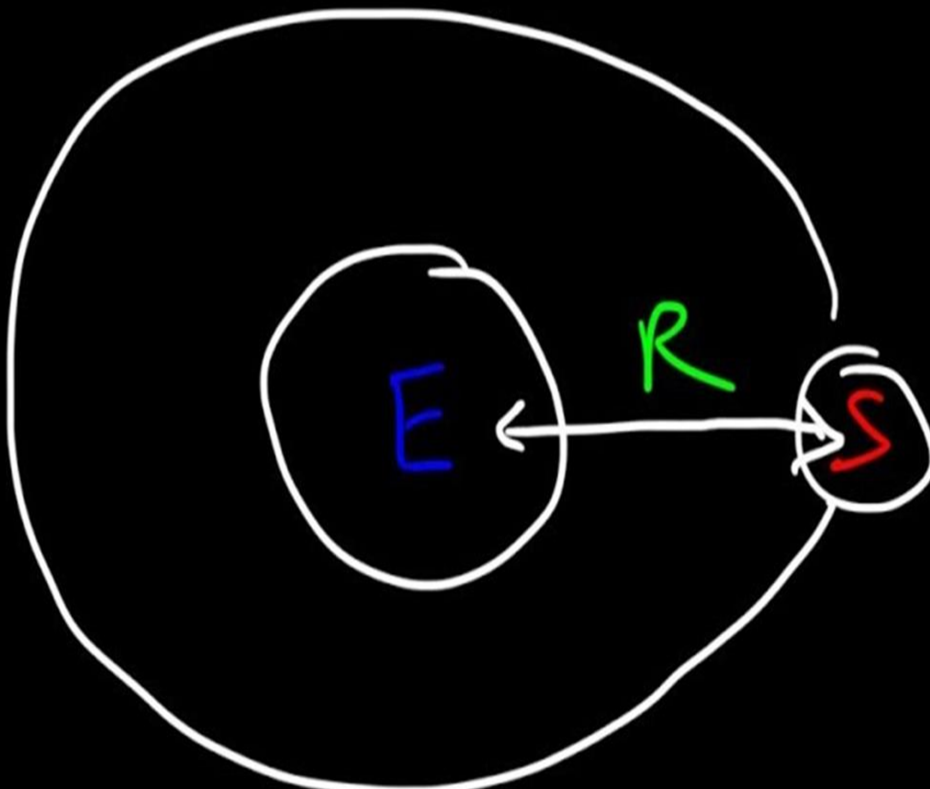


Figure 3.16 Orbital velocity

Satellite Speed



$$V = \sqrt{\frac{GM}{R}}$$

$$\boxed{V = \frac{2\pi R}{T}}$$